

A PROPOSED NEW STANDARD
FOR
HIGH VOLTAGE MEASUREMENTS

Thesis by
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Summary

It is suggested that the electrostatic force of attraction between spheres, rather than the sparking voltage, be adopted as the standard method of measuring high voltage. This proposal is based on the following investigations and results:

1. The sparking voltage of 50 centimeter standard testing spheres varies considerably even when corrections are made for changing air density. At larger spacings, the gap is influenced by the unavoidable presence of ground planes.

2. Accurate force measurements made over a period of several months gave a mean sparking voltage curve which differs by a few per cent from a newly recommended A.I.E.E. standard curve.

3. Only one correction factor, to account for the presence of extraneous bodies in the laboratory, need be applied to voltage computed from measured forces. It is shown that this factor is small and a technique is described for its precise computation.

4. Computations have been made for the effect of the spheres' shanks and the laboratory floor and walls on the attractive force between spheres.

The main results of this research have been published in a paper, "The Sparkless Sphere Gap Voltmeter II", Electrical Engineering, Vol. 55, No. 5, by R. W. Sorensen and the author. This paper has been scheduled for discussion at the A.I.E.E. 1936 summer convention.

INTRODUCTION

For some years there has been a demand for more accurate high potential measurement. Need for a primary standard and dis-satisfaction with present standards were so emphasized by lengthy discussions at the 1934 Summer Convention of the American Institute of Electrical Engineers that this problem was made a part of the research program of the California Institute of Technology High Voltage Laboratory.

Accordingly, Dr. Jesse E. Hobson^{1*} and the author, working under Professor R. W. Sorensen, commenced work on what is now termed the "Sparkless Sphere-Gap Voltmeter"². The pioneering work done by Dr. Hobson is completely described by him in his thesis for the Doctorate degree submitted in 1935 and entitled, "High Voltage Precision Measurements". The results of this research may also be found in a paper, "The Sparkless Sphere-Gap Voltmeter", by Professor R. W. Sorensen, J. E. Hobson, and the author of this thesis². (Because this paper includes a description of some of the apparatus used in the research reported in this thesis as well as some curves and tables used in computations, it has been incorporated into this thesis as Appendix A.)

The preliminary work on the Sparkless Sphere-Gap Voltmeter showed it to have promise as a means for the accurate measurement

* For all numbered references, see Bibliography.

of high voltage. It was found to have many advantages over the conventional sparking spheres (see Appendix A). The question as to whether it could be termed an absolute standard was still to be investigated, for the computation of voltage from a measured force assumed isolated spheres, whereas actually the floor and walls of the laboratory and the leads and supporting structure were known to influence the relation between force and voltage. An analysis of means whereby the effect of floors, walls and shanks, etc. could be determined and possibly a quantitative evaluation of the disturbance in terms of a correction factor to be applied to the computations were desired.

Recognition of inaccuracies in the present A.I.E.E. Standard Sparking Curves for spheres from 25 to 200 centimeters brought a request from the Measurements and Standards Committee of the A.I.E.E. for a recalibration of the sphere gaps. In Appendix B, the means for arriving at both the old sparking curves and those of recent investigators are described briefly. It is seen that there was little reason to expect the previous curves to be correct and little more reason to expect accuracy on the part of recent investigators; their results differ by several per cent. Thus, a careful measurement of the sparking voltage for a standard gap by use of the sparkless spheres promised to be of great value and was undertaken.

The sparking voltage between spheres of definite size and gap spacing is known to depend on many factors. If sparking

spheres are to continue to serve as a means for measuring high potentials then it is necessary to ascertain the relations between the sparking voltage and the factors which influence its value. Because the force readings are known to be independent of most of these factors, such as changing atmospheric conditions, pertinent data on the behavior of a gap over a period of several months could be obtained by repeated measurements of sparking voltages. Further, it was deemed of value to investigate quite completely the effect of the proximity of ground planes on the sparking voltage of a sphere gap.

MEASUREMENT OF SPARKING VOLTAGE
FOR FIFTY CENTIMETER SPHERES

Description of Apparatus

Voltage was obtained from the one million volt cascade transformer of the California Institute of Technology.³ The effective value of the voltage wave delivered to the spheres through a water hose resistor by the transformer set was known to be 99.25 per cent that of a true sine wave having the same crest value. (See Figure 5, Appendix A.)

The force meter is completely described in Appendix A and none but small changes were made in applying the meter to this series of measurements. A simple damper consisting of a vane about one-foot square suspended in oil was installed. The vane was mounted on the shank of the moving sphere near the cathetometer table. However this damper was removed after a series of tests showed that its beneficial effect in reducing oscillations was more than offset by the increase in time needed for the moving sphere to respond to a very small force. Also, it was attempted to improve the sensitivity of the force-measuring device by replacing the bicycle wheel by a device having even less friction and permitting a multiplication of the forces to be read. A scheme investigated was the simple one of making the end of the shank (to which previously was fastened the cord passing over the bicycle wheel - Figure 3 of Appendix A) support

one of two very thin equal length wires carrying the balancing weight, the other wire being held by a stationary support. The length of the wires and the distance between moving and fixed supports being known, the horizontal pull on the spheres for a given weight is also known. In practice it was found that the small changing vertical component acting on the shank of the sphere due to the balancing weight actually caused sufficient deflection to disturb the zero reading on the cathetometer. To eliminate this error would have introduced more friction, so the original force-measuring device was again installed.

The fifty centimeter spheres are spun aluminum, standard testing spheres purchased from the General Electric Company. A special frame (Figure 1) was constructed for them, using paraffined maple and pine and with clearances as recommended by A.I.E.E. standards for high voltage testing. The shank of the lower, grounded sphere was equipped with an indicator, adjustable for zero setting, which enabled the reading of gap spacing on a parallel scale.

To obtain the very large number of spark-overs desired to give dependable results, it was necessary to protect the spheres against pitting. Experience had shown that a water rheostat to limit the current after spark-over was entirely unsatisfactory because of the difficulty of keeping the rheostat in operating condition after it had suffered a number of spark-overs, and of



Figure 1.

designing it for the high voltage conditions in the first place; also, even with a megohm or more of resistance, the spheres would pit badly before the voltage could be removed by the breaker operating from the remote controls. Through a gift from the Kelman Electric & Mfg. Company, a special quick opening breaker was installed in the primaries of the million volt set. Several current transformer coils were made available so that the trip circuit could be energized by an arc at any voltage, even with the one megohm resistance in series.

The dimensions of the laboratory and the arrangement of equipment are shown in Figure 2. The clearances are seen to be quite large. S₁ is the sparkless 100 centimeter sphere-gap voltmeter and S₂ is the standard 50 centimeter sphere-gap. The laboratory is 64 feet wide with the test gaps approximately along the center of the building; all dimensions shown are in feet. The parallel connection of gaps as shown was unbroken except for a minor set of experiments to determine whether the presence of the sparkless spheres would influence the readings of the sparking spheres.

Calibration Procedure

The calibration procedure was as follows:- The 50 centimeter sphere-gap was set at some desired spacing and a large number of spark-over tests made for each setting, the voltage on the tertiary or voltmeter coil of the first transformer being noted at the instant of spark-over. The voltage was increased from zero to sparking value at as fast a rate as possible consistent with the ability to read the tertiary voltmeter accurately. Slower rates of voltage rise are productive of greater variations in spark-over voltage for any given gap setting, since if the voltage is held just below normal spark-over value for a long enough time, the gap it seems, will finally spark over.

These tests were followed by and interspersed with many tests to determine the relation between tertiary voltmeter readings and

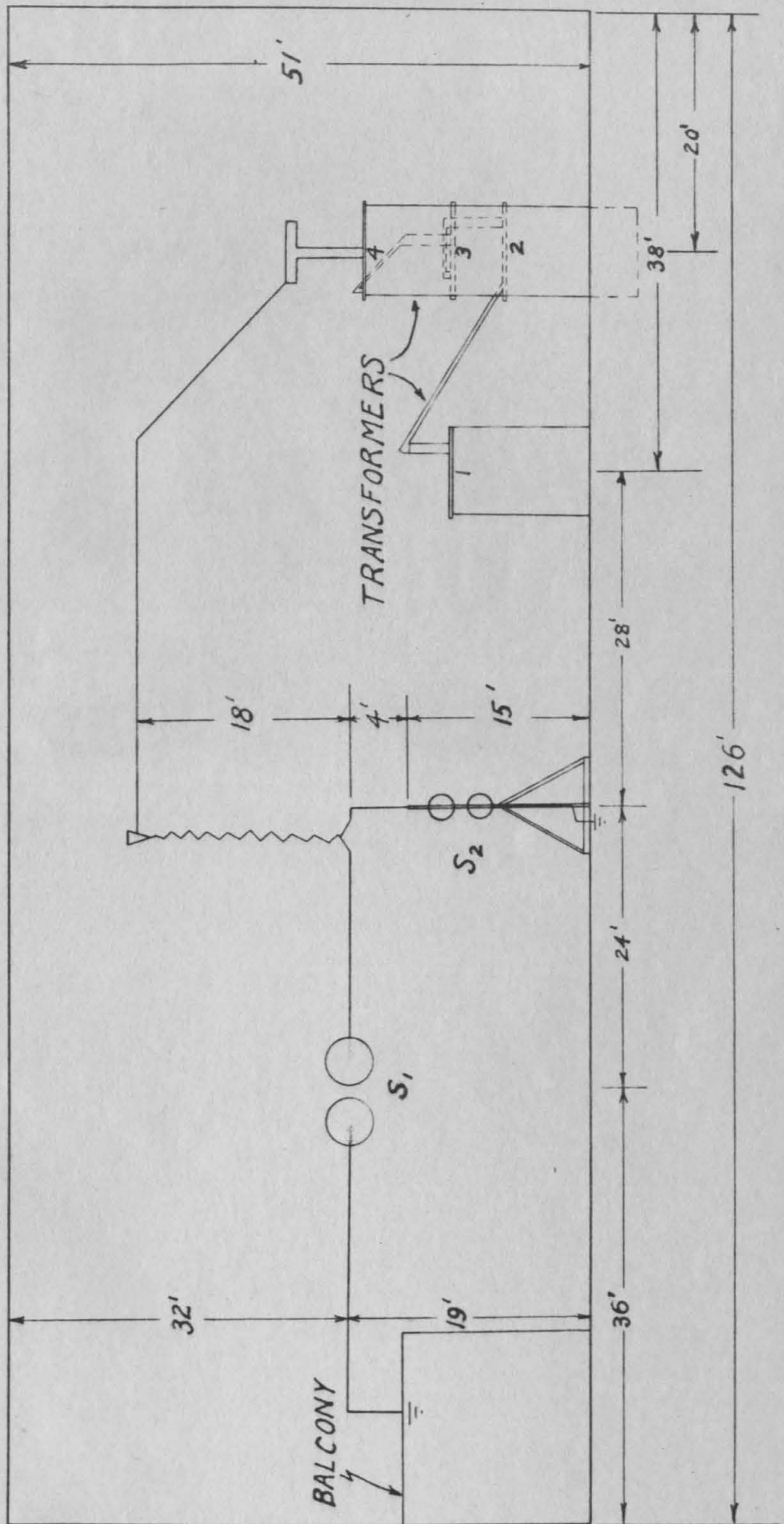


Fig. 2

force measurements on the 100 centimeter spheres. Thus the tertiary voltmeter readings were made to serve wholly as comparative readings, making unnecessary a determination of the relation between tertiary voltage and voltage of the transformer. Each test period consisted of two sets of readings, one of tertiary voltages at which sparking occurred on the fifty centimeter spheres for a small range of spacings, the other of the forces between the sparkless spheres at known spacings and over the same range of voltage as in the first set of readings, as indicated by the tertiary voltmeter. During each test period, all connections and equipment were undisturbed except for a special minor series of tests to be described. A preliminary set of experiments showed that the voltage may vary slowly and considerably during the time necessary to obtain a balance with the force-measuring device; these experiments also showed that the moving sphere was sufficiently sensitive to determine whether the voltage was changing or constant. Accordingly, a signal system was used so that the tertiary voltmeter was read always at the instant of balance.

To insure a maximum of force, the spacing on the sparkless spheres was kept at a minimum for each voltage range. The meter was adjusted for zero setting (see Appendix A) at the beginning of each test period and returned often to zero spacing to disclose any shifting of the supports. Atmospheric pressure and temperature

were observed and all readings of sparking voltage were corrected to 25 degrees Centigrade and 760 millimeters barometric pressure⁴. From the known force and sparkless sphere-gap spacing, the value of voltage was computed using the equation $V = 9405 \sqrt{\frac{F}{S}}$ and the values of S given in Table III of Appendix A.

Typical test data is given in Table I. The relation between tertiary voltmeter reading and voltage computed from the measured force was taken from curves (Figures 3 and 4) drawn through the mean of points determined during many test periods and by different observers. A consideration of the possible errors made in obtaining the points on these curves indicated this procedure to be better than that of obtaining the sparking voltage for any one period of tests from a curve between force and tertiary voltage obtained during that same period.

Tests were for the most part made with the sparking gap in normal operating position, the axis of the gap vertical and the center of the upper and fixed sphere about 275 centimeters or 5-1/2 diameters above the floor. Two sets of tests were made with the gap horizontal, one at the same height - about 5 diameters above the floor - and the other at a distance of 3 diameters above the floor. Also, a few readings were taken with the gap vertical and over 10 diameters above the floor. A minor series of readings of tertiary voltage at sparking was made with the connection to the sparkless spheres removed to determine the influence of this change.

Table I
Typical Test Data

October 29, 1935

Sparkling Spheres

Zero setting 7.72 cm.

Barometer b 740 m.m.

Temperature t 71° F

$$\delta = \frac{3.92 \text{ b}_{m.m.}}{273 + t_{cent.}}$$

$$V_2 = \frac{V_1}{\delta}$$

Scale cm.	Gap cm.	Tertiary Volts Meter A	V ₁ from Figure 4	V ₂ Corrected Voltage
11.72	4	21.3	72.7 K.V.	74 K.V.
"	"	21.6	73.6	75
"	"	22.0	75.2	76.5
12.72	5	27.05	93.5	95
"	"	26.7	92.0	93.5

Sparkless Spheres

Zero Setting 7.7 cm.

Scale cm.	Gap cm.	Tertiary Volts Meter A	Force grams	Voltage Computed
12.7	5.0	22.0	65	71.1 K.V.
"	"	23.8	83	80.5
"	"	26.3	100	88.5

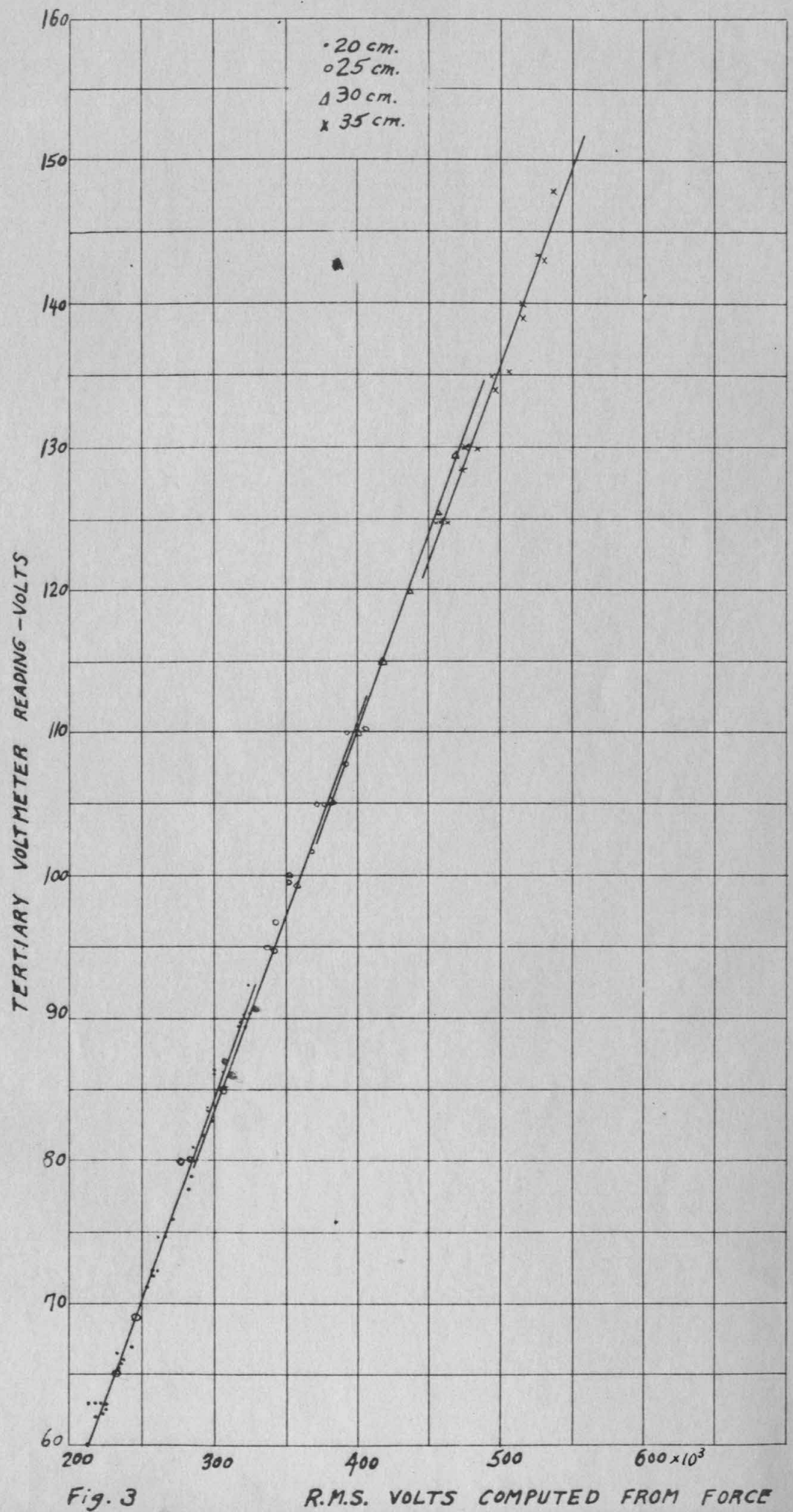


Fig. 3

R.M.S. VOLTS COMPUTED FROM FORCE

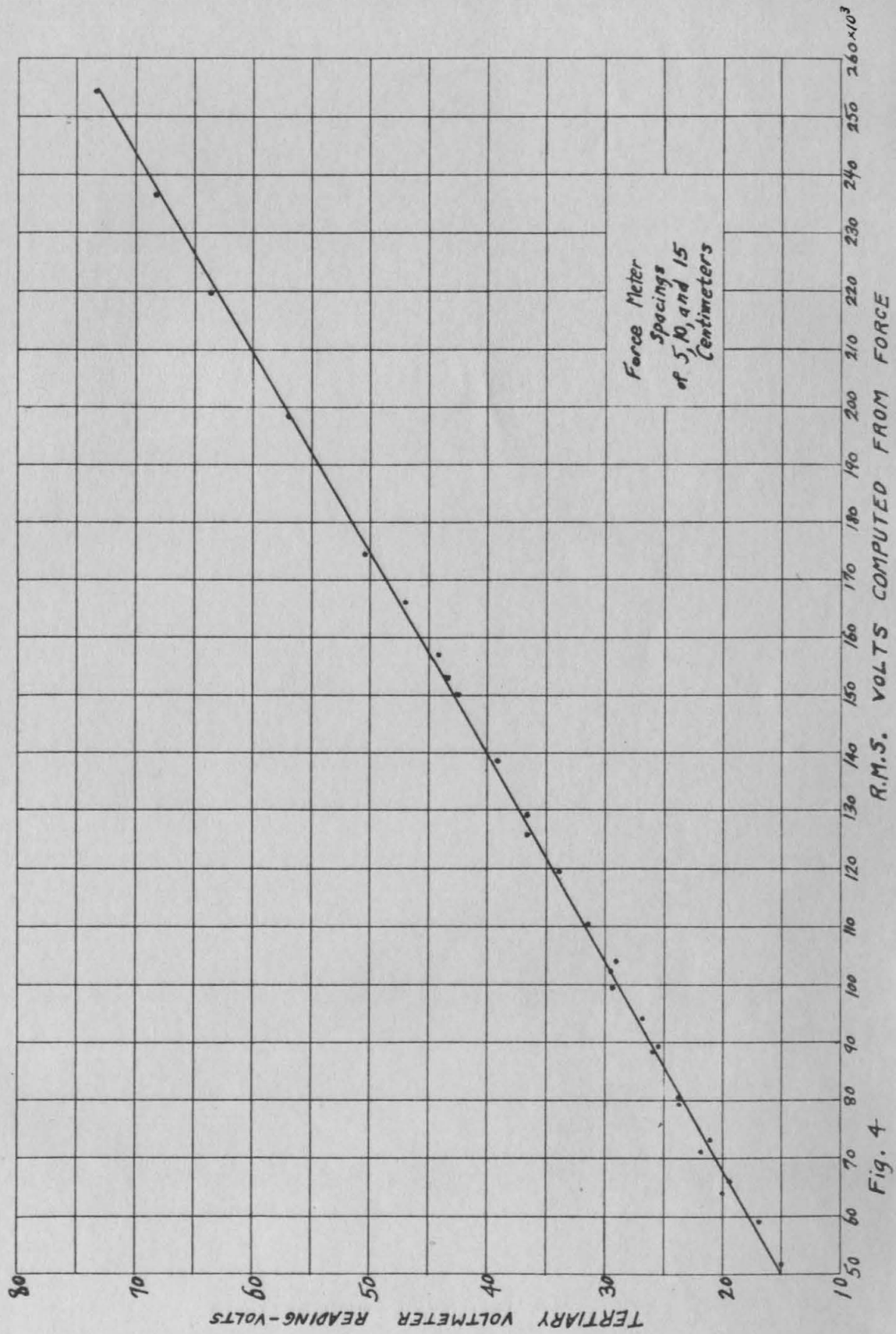


Fig. 4

Results of Spark-over Measurements

The results of the tests described above for the sparking spheres in the normal vertical position with the upper sphere at about $5\text{-}1/2$ diameters above the floor and the lower sphere grounded are best seen from Figure 5. In this figure, the test points obtained are plotted so that the ordinate represents the differences in crest kilovolts between the voltages indicated by the points and the voltages at which, according to a proposed new A.I.E.E. standard sparking curve (Appendix B), the gap for a given setting should spark over. Values obtained by other investigators are also shown. Figure 6 gives the usual form of spark-over curve drawn through the mean of the points in Figure 5. In Figure 6, curve A is the proposed new A.I.E.E. standard curve, B is from Bellaschi⁵, C is the curve through the mean of points in Figure 5, M is from Meador⁶, and P is from Peck⁴. At the higher spacings, the gap was found to require several conditioning spark-overs before indicating the ability to give consistent readings. Except for these conditioning readings and a few others quite evidently in error, all readings have been plotted. Thus, the total spread of points shown for any given spacing is not that found during any one test period, but is that which may be expected for tests extending over several months under varying conditions as to temperature, humidity, and barometric pressure even with the usual corrections for air density variation.

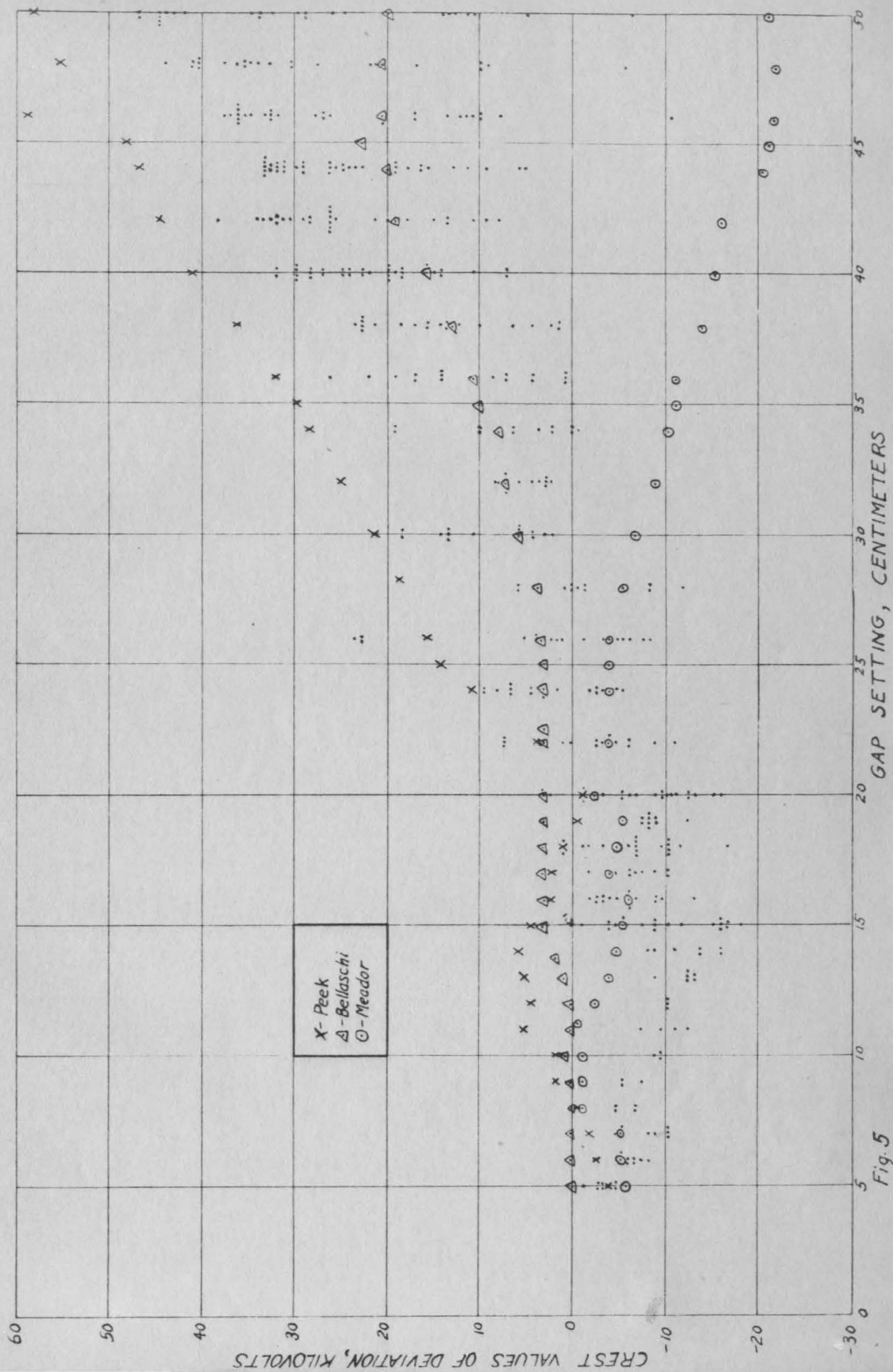


Fig. 5

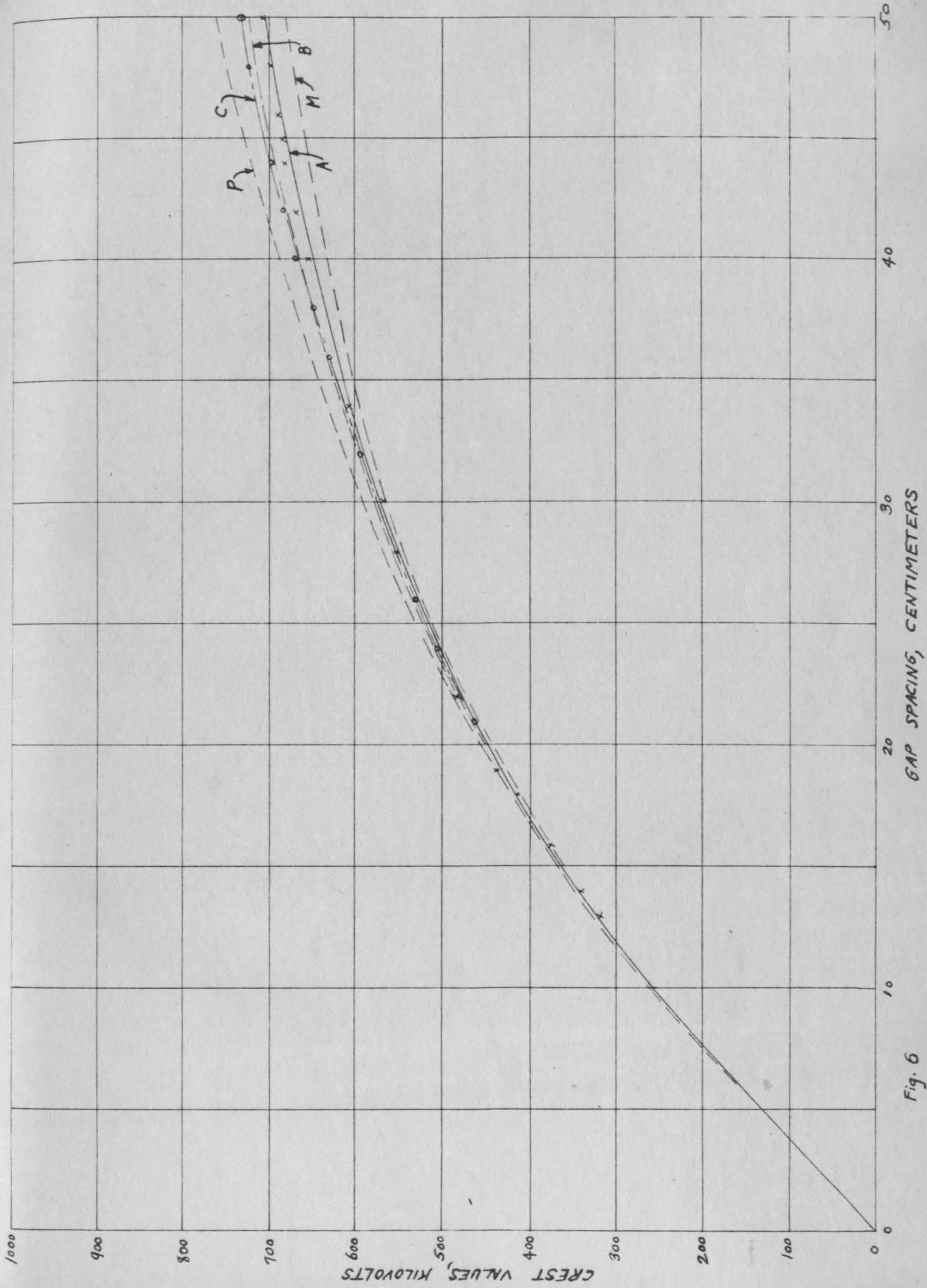


Fig. 6

In Figure 7 are plotted the deviations of test points from the proposed A.I.E.E. sparking curve for the case of spheres horizontal about $5\frac{1}{2}$ diameters above the floor. Here it is noted that the test points follow those for the vertical setting for the same clearance to ground until about half diameter is reached; at larger spacings, the gap when horizontal sparks over at voltages somewhat lower than for the same spacings with the gap vertical. The mean of the points in Figure 7 are plotted with crosses on the curves of Figure 6. As would be expected, the large spread of points appears for the horizontal as well as the vertical setting, except for the very highest spacings. This may be because at these large spacings the ground plane is sufficiently active as a third electrode to cause sparking more consistently along a certain definite path. In the vertical set-up the arc was observed to vary over quite an area for the larger spacings.

With the gap horizontal and only 3 diameters above the floor, spark-over occurred for all spacings at voltages much lower than in the two previous cases. That the gap was greatly influenced by the presence of the ground plane could be seen from the shift of the position where the arc struck on the surfaces of the spheres. At a spacing of 40 centimeters, the voltage in this position was less than 90 per cent of the voltage required for sparking with normal clearance.

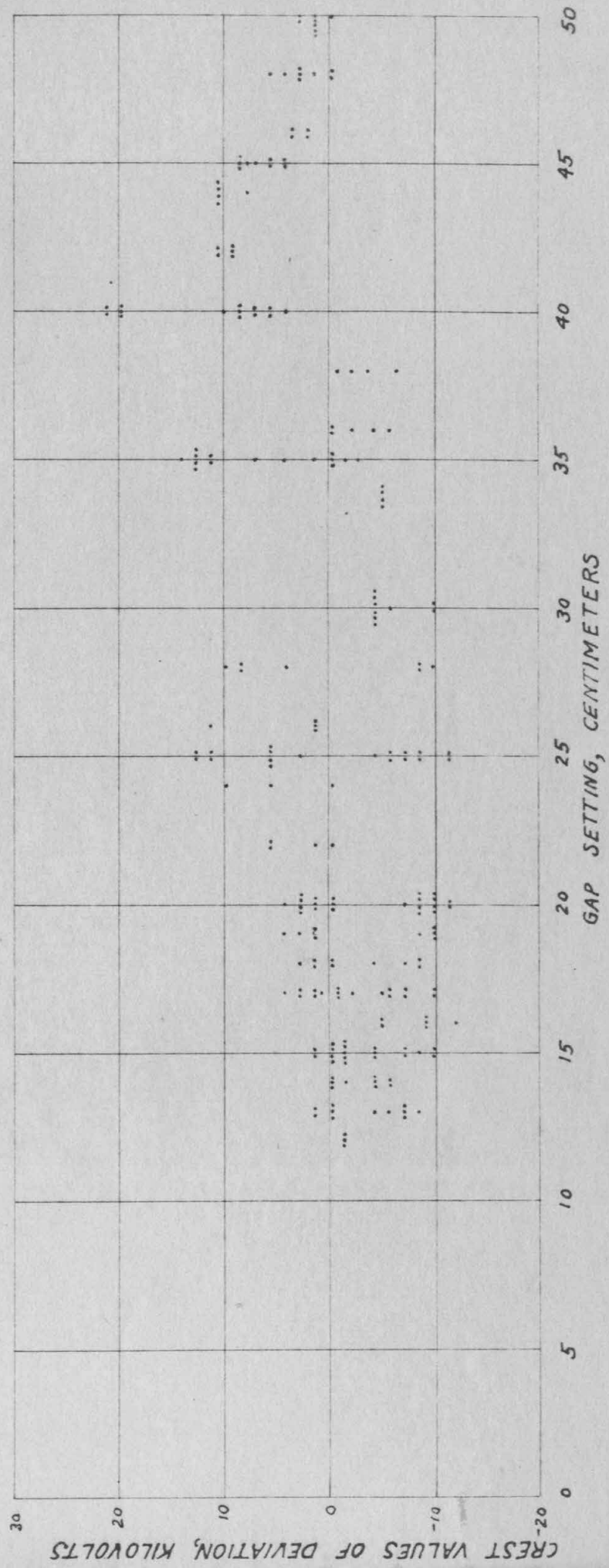


Fig. 7

The set of tests made on the gap when vertical and 10 diameters above the floor (with even more clearance to the ceiling) gave readings almost identical with those for the normal vertical position up to one-half diameter spacing; for greater spacings the voltage required to produce sparking was found to be greater than that in the normal vertical position. The difference became as much as 5 or 6% at nearly full diameter spacing. This difference was probably due not only to the diminished effect of the floor, but also to the increased effect of the ceiling. In the normal position the lines of force concentrate toward the grounded sphere due to the additional attraction of the ground, while at the higher position there are probably fewer lines coming from the hot sphere, due to the decreased capacity, and some of these lines end on the ceiling instead of on the grounded sphere.

When the sparkless spheres were disconnected, the tertiary voltmeter read a lower voltage when spark-over occurred for any given spacing of the sparking spheres than it indicated with both gaps, as normally used in these tests, in parallel. The effect of this change of connections should have been to decrease the current through the series water rheostat. However, this may not have caused an appreciable increase of voltage at the spheres since the current was strongly leading. Also, a decrease in the leading current probably caused a drop in the terminal voltage of the transformer which would buck the effect of the diminished drop

in the rheostat. It is not possible, without additional data, to say how much of the 2 or 3% change in tertiary voltage reading at sparking was due to a change in the relation of tertiary volts to volts at the spheres; but very probably some of this effect was due to the action of the electrostatic field coming from the sparkless spheres.

The following conclusions have been reached by a survey of the results of the measurement of sparking voltages:

1. The proposed new A.I.E.E. standard sparking curve for the 50 centimeter spheres indicates voltages which are too low for the larger gap spacings and too high for the smaller spacings.

2. The influence of the floor and walls of the laboratory on the sparking voltage is very small for any position when the gaps are of the order of 6 diameters from the ground plane and the gap spacings are less than three-fourths full diameter.

3. The sphere spark-gap voltmeter becomes increasingly inconsistent as the gap spacings exceed three-fourths full diameter. The spark-over voltage is greatly influenced by the proximity of ground planes at these higher spacings, the influence being observed even at clearances from 6 to 10 diameters.

4. Even when corrections are made for barometric pressure and temperature changes, the sphere spark-gap will not show the same sparking voltage at all times, the differences in readings being as much as 6 and 7 per cent and averaging a few per cent at all spacings.

Accuracy of Measurement of Sparking Voltages

The accuracy of sparking voltage determination by means of the sparkless sphere gap is a function of the accuracy possible in determining:

1. Tertiary voltmeter reading
2. Sphere gap spacings
3. Force
4. Spacing factor S used in the equation $F = S V^2$

where F is the force and V is the voltage impressed on the spheres.

The accuracy of single readings for items 1 to 3 was of the order of .2 per cent for tertiary voltmeter readings, 1/2 millimeter for readings of sphere gap settings, and within a gram for force readings, the forces ranging from 100 to over 350 grams. From Figures 3 and 4, it is possible to make a satisfactory estimate of the overall accuracy of items 1 to 3. To obtain each point plotted on Figure 3, a reading had to be taken of the tertiary voltmeter, the zero or contact position of the sparkless spheres, the gap setting of the sparkless spheres, and the force. The average deviation of these points from the mean was only about 1/2 per cent. The kilovolts for any tertiary voltmeter reading observed at spark-over of the 50 centimeter spheres was taken always from these curves. Now if item 4 is considered to cause no error, the error in each spark-over point plotted in

Figures 5, 6, and 7 is the sum of an error in a single tertiary voltmeter reading, a gap setting on the fifty centimeter spheres - both of these in the tenths of a per cent - and an error in the relation between kilovolts and tertiary volts obtained from Figures 3 and 4. The latter error must be far less than one-half per cent. Thus, aside from item 4, the test points should not be off more than about $1/2$ per cent as a conservative estimate.

The fourth item, spacing factor S, can be calculated accurately for isolated spheres (see Appendix A). True isolated spheres are, of course, impossible and for actual conditions one must, as in the case of standard sphere gap measurements, take account of supporting shanks and adjacent bodies. The accuracy with which the spacing factor for isolated spheres may be applied to the spark-over voltage measurements is dealt with in later sections of this thesis. The results of the investigations described there indicate a correction not exceeding 1.5 per cent to be applied to the test points of Figures 5, 6, 7 in the region from 40 to 50 centimeters spacing; not over .5 or .6 per cent in the region from 30 to 38 centimeters spacing; no correction below 30 centimeters spacing. The direction of the correction is such as to decrease the values of voltages plotted on these curves.

Preliminary Investigations of Possible Means
for Determining the Value of the
Sparkless Sphere Gap Voltmeter as an Absolute Standard

Were it not for the laboratory floor and walls and the shanks on the spheres, the sparkless sphere gap voltmeter could immediately be termed an absolute instrument, since the method of inverted images allows an accurate computation to be made of the electrostatic force of attraction between the spheres for any given applied voltage. (Appendix A) The influence of shanks and adjacent bodies on the attractive force between the spheres may be determined in theory by experiment or by calculation, but in actual operation both methods present difficulties. Investigations of both a theoretical and experimental nature were made before choosing the means later described for obtaining the values of corrections to be applied to computed sparking voltages. The methods examined were as follows:

1. A measurement of the rate of change of capacitance of the spheres with respect to distance between them would give an overall calibration of the instrument taking into account every possible disturbing influence. This is because the spacing factor relating the force to voltage is numerically equal to one-half the rate of change of capacitance, as may be seen from a consideration of the energy in the electrostatic field. Using

Maxwell coefficients of capacity:

$$E = \frac{1}{2} C_{s1} V_1^2 + C_m V_1 V_2 + \frac{1}{2} C_{s2} V_2^2$$

where E is the energy, V_1 and V_2 are the potentials of the spheres, C_{s1} and C_{s2} self-capacitances, and C_m is the mutual capacitance of each sphere. C_s is the charge existing on one sphere when it is at unit potential and the other sphere is grounded; C_m is the charge on either sphere when it is grounded and the other sphere is at unit potential.

The force of attraction (taken as positive for attraction, negative for repulsion) is given by differentiating the expression for energy with respect to distance between the sphere centers.

$$F = \frac{1}{2} \frac{\partial C_{s1}}{\partial x} V_1^2 + \frac{\partial C_m}{\partial x} V_1 V_2 + \frac{1}{2} \frac{\partial C_{s2}}{\partial x} V_2^2$$

For our laboratory set-up, V_2 is zero and

$$F = \frac{1}{2} \frac{\partial C_{s1}}{\partial x} V_1^2$$

A resonant circuit was set up using a frequency of a few thousand cycles. Readings of the standard condenser in the circuit were made for spacings of every few centimeters of the sparkless sphere gap. It was found by repeating the curves thus obtained with the leads going to the hot sphere in different positions that the various curves did not check very closely. Some of this apparent distortion of the curves was undoubtedly in the

the inability to set the spheres to the same spacing each time. A gap spacing measurement, accurate enough for ordinary voltage measurements was not sufficiently precise, by far, to give a curve whose slope had to be found to within a few tenths of one per cent accuracy in order to be of value. Also, the capacity being of the order of 50 to 100 micro-micro-farads, the differences of capacity noted on the standard condenser were too small for the necessary precision.

It was concluded that this method would be quite difficult to extend to the degree of accuracy required, though the design of a special condenser reading differences to a small part of a micro-micro-farad and used in a bridge circuit of a type in which only the differences of capacity would be of significance, certainly would yield better results than those obtained. In any case, the stray capacities of the long leads would still be present and great care would have to be taken to see that the change of this stray capacity as the gap setting is changed would not be enough to influence the readings.

2. Another attack on the problem of measuring rates of change of capacity was made in considering a ballistic galvanometer, direct current voltage circuit in which charge on the spheres and voltage would be determinable. Here, because of the small capacity to be measured, an ordinary galvanometer would give too small a deflection if the charging voltage of the spheres were

low enough to be measured with precision. It was necessary therefore to investigate the possibility of charging the spheres to a voltage in the neighborhood of 100,000 volts D.C. from a kenotron and measuring the charge, which was then suitable in magnitude for the available galvanometer.

The voltage would be measured by measuring the force. Then the capacity would be given by:

$$C = \frac{Q}{V} = \frac{Q}{\sqrt{\frac{F}{\frac{1}{2} \frac{\partial C}{\partial x}}}}$$

A curve of capacity against spacing, the capacity being computed from the above equation using values of $\frac{\partial C}{\partial x}$ for the isolated sphere case, would be obtained. From this curve, $\frac{\partial C}{\partial x}$ would be extracted and a new series of calculations for C made, using these new values of $\frac{\partial C}{\partial x}$. This process could be continued until the values of $\frac{\partial C}{\partial x}$ taken from the curve agreed with those used in arriving at the curve.

The voltage was to be maintained at a steady value by charging a high voltage condenser bank of large capacity from the kenotron through a very high resistance.

Another procedure using the same apparatus and a simple device designed to move the sphere suddenly a small known distance was also considered at the same time. With the hot sphere already charged to the high D.C. potential, the galvanometer would read difference of charge as the sphere was deflected.

Then

$$\frac{\Delta Q}{\Delta x} = \frac{\Delta C}{\Delta x} V$$

and with very small displacements,

$$\frac{\Delta Q}{\Delta x} = \frac{\partial C}{\partial x} V = \frac{\partial C}{\partial x} \sqrt{\frac{F}{2 \frac{\partial C}{\partial x}}}$$

$$\text{or } \frac{\partial C}{\partial x} = \left(\frac{\Delta Q}{\Delta x} \right)^2 \frac{1}{2F} = \frac{\partial C}{\partial x}$$

(Note that this gives actually the value of $\frac{\partial C}{\partial x}$ desired, which is the rate of change of self-capacity of the hot sphere with respect to displacement of the grounded sphere. The previously described measurements give the rate of change of self capacitance of the hot sphere as the hot sphere is moved. The difference should be very small.)

Though the difficulty of completely shielding the galvanometer with both terminals at high potential was overcome, the leakage current through the galvanometer was so high as to make any additional deflection due to a change of charge through the element very small by comparison. This difficulty could not be eliminated, except perhaps by an elaborate insulating system to support the high voltage sphere in place of its present support.

3. It was believed that valuable information could more easily be obtained on a model of the voltmeter to a small scale.

With the model spheres, comparisons of forces due to a constant applied voltage with and without the presence of ground planes can be made. Some preliminary work was done on 12.5 centimeter spheres and movable grounded screens. This work indicated possibilities, but also that the time and care required to obtain the quantitative results needed were sufficient to warrant a separate thesis for the doctorate. Accordingly, R. B. Vaile, Jr. undertook the problem and the results of his investigations will be found embodied in his thesis.

4. Computations by the method of images for the effect of floors and walls of the laboratory were investigated in a very rough way (Appendix A). The few results obtained were of almost unknown accuracy, since many doubtful approximations were made. It was known however that with sufficient labor in performing tedious calculations, this method would lead to dependable results. After the preliminary investigations of all the methods described, the image computations were thought to offer the best possibilities for complete analysis.

5. A scheme which suggests itself for taking account of the effect of shanks on the force between spheres, is to place a line charge along the shank of each sphere. These line charges would, of course, have to be of varying density such that when imaged into both spheres, leaving them at the proper potentials, the potentials on the shanks due to the original image charges

in the spheres plus the two line charges and their images would be the same as on the sphere to which the particular shank is attached. To determine the possibility of arriving at the value of the line charge densities to satisfy these conditions, a simpler case was considered. A line charge was placed along the axis of a cylindrical shank of finite length and the potential at any point on the shank due to this line charge of varying density, was equated to a constant less the potential due to a point charge placed some distance away along the extended axis of the shank. The resulting integral equation was solved by Professor Bateman. The simplest form of the solution for the density of the line charge was a complicated Fourier series of doubtful convergence.

The line charges may be replaced by a series of point charges giving a shank with "humps". This was found to be practical for computation. A calculation for the effect of such shanks at two spacings is presented in a later section of this thesis.

Image Computations for the Effect of
Ground Planes and Shanks on the Force between Spheres

The first step in the computation of the effect of ground planes on the force between spheres was a study of the relative order of magnitude of the various new image charges created by a single pair of image spheres. These image spheres were located with their line of centers parallel to the line of centers of the original spheres and a distance of twelve diameters away. This case corresponds to the laboratory floor, six diameters from the spheres.

For this study the spheres were drawn out on a sheet of paper (12 feet by 4 feet) large enough to allow image charges and their positions to be placed on the plot as they were found. Then the procedure was as follows:

1. Principal charges were found for the case of two isolated 100 centimeter spheres at 25 centimeters spacing. (See Appendix A for the method.) After the fourth or fifth image had been obtained in each sphere, it was noted that the remaining images occurred always at the same position. (The distance of the image from the center is $\frac{r^2}{f}$ where r is the sphere radius and f is the distance to sphere center of the charge to be imaged. The image position was always 25 centimeters from the center of the sphere. This makes $f = 125 - 25 = 100$. Then since $r = 50$,

$\frac{f^2}{f} = 25$ for the next image, etc.) By simply summing a geometrical series, one can obtain the principal charges and their positions to a high degree of precision in but a few operations. Using the principal charges thus obtained, the force between spheres was calculated by slide-rule to be

$$F = .174 V^2$$

while Kelvin gives $F = .17432 V^2$

2. These principal charges were placed also in the image spheres below the floor plane. These four series of principal charges (one series in each of the four spheres) were designated by a letter P with appropriate subscripts. This subscript system made the "book-keeping" of the images possible and thus lessened the difficulty of checking calculations.

3. The two P series in the image spheres below the ground plane were imaged into each of the original spheres; the two P series in each of the original spheres were imaged into each of the two image spheres below the ground plane. The new images thus created were termed the S images (with appropriate subscripts to designate the sphere which contained the particular images).

4. The S charges were imaged into all the spheres, giving the T set of charges; the T set gave rise to the U set, which gave rise to the V set, which gave rise to the W set.

The charges making up the W set were very minute and were altered to take account of the sum of all charges beyond the W set.

5. The additional force between spheres due to (1) the presence of the P images in the image spheres and the S, and T image charges was computed. Then the additional force due to (2) the U, then (3) the V, then (4) the W sets of charges was computed. The ratio of the separate contributions was (1) 7.5, (2) 2.4, (3) -2.5, (4) 2.1, and the total contribution was $\frac{9.5}{697.2}$ or about 1.35% of the force due to the original principal charges in the original spheres. It is important to note that it was not attempted to recalculate the total force between spheres when in the presence of a ground plane. This would have required a very high degree of accuracy in placing and determining the new image charges, for to detect a difference of 1.35% would have meant that the new total force must be computed to better than one per cent. However, by computing the additional contribution as was done here, the value 1.35 should be correct to slide-rule accuracy. Because of the differences that enter into the calculations, it was believed that the value of 1.35 simply showed the correction to lie definitely between 1 and 1-1/2 per cent.

This first analysis disclosed certain facts which made possible quicker and more accurate computations. The additional contribution to force caused by the images which spring up due

to the presence of a ground plane may be divided into two parts:

- (1) the additional force due to new images in the original sphere;
- (2) the direct new force due to all the charges in the image spheres.

It is easy to estimate the value of factor (2) by simply considering the principal or P charges in the image spheres acting on the principal or P charges in the original spheres. (The value of factor (2) would change only by 1 or $1\frac{1}{2}$ per cent, if charges S, T, etc. were also considered.) By thus estimating factor (2), it was found that it was but of the order of one per cent of factor (1) for the 6 diameter to ground plane case. Thus if item (1) is computed accurately, the entire effect of the ground plane is known accurately.

A consideration of the quantities influencing factor (1) shows that the new charges arising in the original spheres because of the presence of the ground plane consist of one group very close to the center of each sphere and another group not at the center, but which arises from those which are at the center. In Fig. 8, the original spheres and the image spheres are shown. (The principal charges are not present in this diagram.) The new images which arise in each sphere because of the presence of the ground plane are indicated by the dots which are intended only to illustrate how the new charges actually do lie either very close to the center or else appear (in the course of imaging the "center" charges) at some position from 20 to 25 centimeters from the center.

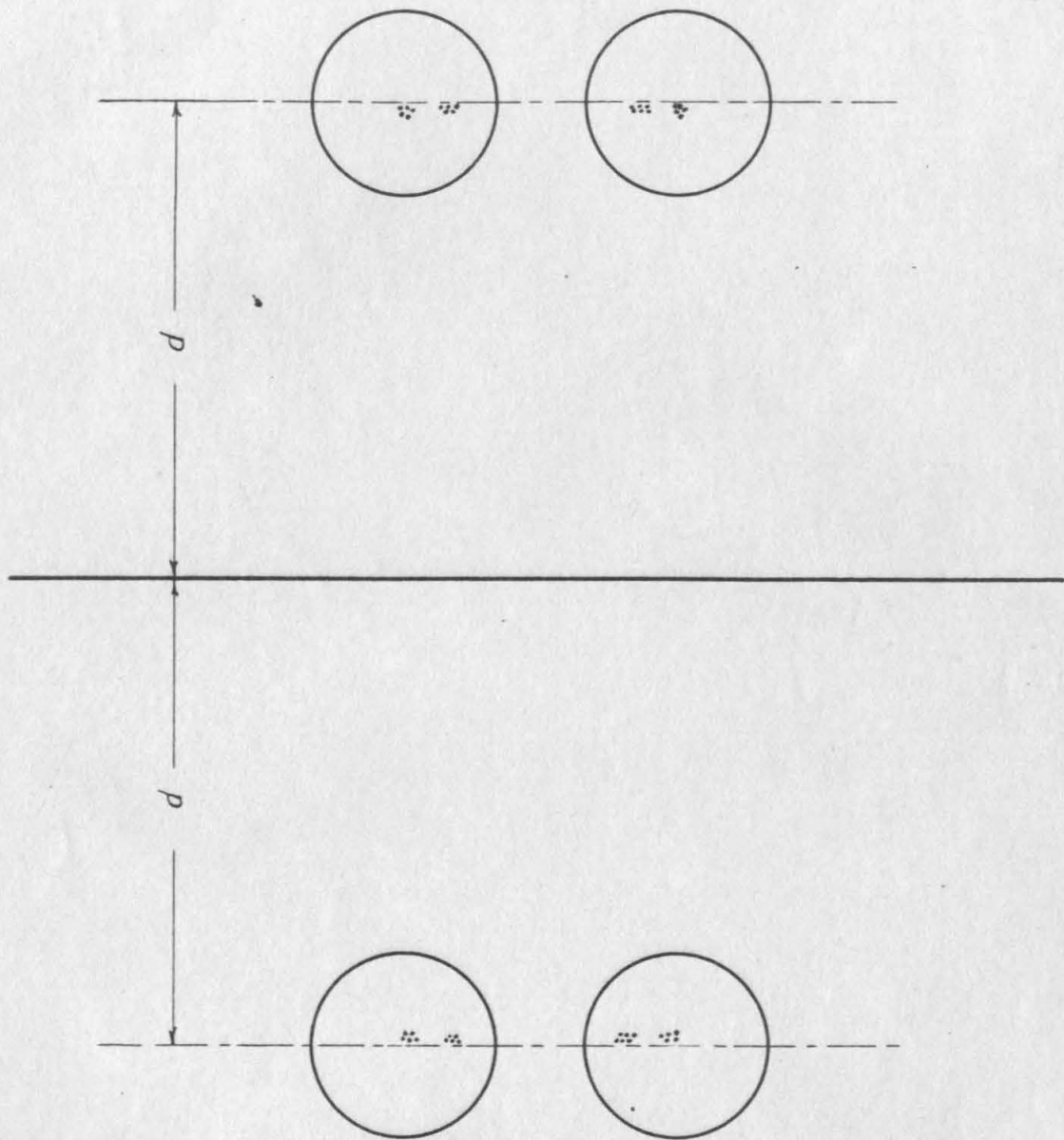


Fig.8

Now, the "center" charges are very close to the center; they arise, without exception, to image out all (including the principal charges not shown in the figure) the charges in the two lower image spheres. Since the lower spheres are 12 diameters or 24 radii from the original spheres, the distance that the "center" charges are actually displaced from the center is $1/24$ times the radius (according to the image formulas, Appendix A) or about 2 centimeters. Furthermore, the direction of this small displacement is almost perpendicular to the line of centers. As a result, the charges which appear because of these slightly displaced "center" charges are precisely the same as they would have been had the "center" charges been exactly at the center to a fraction of one per cent. Thus:

$$\frac{1}{24} \times 50 = 2.08 \quad - \quad \text{distance of "center" charge}$$

from center of sphere and on a line almost perpendicular to the line of centers.

$$\sqrt{125^2 + 2.08^2} = 125 \quad - \quad \text{to 3 significant figures the}$$

same distance to the center of the adjacent sphere as would be obtained if the charge were exactly at the center. Consequently, the resulting charges are of the same value and in the same position as that which they would have if the "center" charges were at the center.

Any new charges placed at the center of either of the original spheres give rise to other charges, as stated, at distances

to centers of 20 to 25 centimeters. If the potential of either sphere were to be raised, the new analysis would simply require placing an appropriate charge ($Q = C V$ where C is the capacity of an isolated sphere, equal to its radius) at the center of the sphere, then introducing in each sphere all the image charges that result. But the effect of added potential on each sphere on the force is known. So that it is not necessary to find the values and positions of countless succeeding image charges, and then to multiply all the new charges in the grounded sphere by all the charges in the hot sphere and to divide by the appropriate distance squared, to obtain the disturbing force of attraction. It is simply necessary to determine the new added charge at the center of each sphere, interpret it in terms of potentials $\Delta \bar{V}_1$, and $\Delta \bar{V}_2$ and note that:

$$F_1 = \frac{1}{2} \frac{\partial C_{15}}{\partial x} [V_1^2]$$

$$F_2 = \frac{1}{2} \frac{\partial C_{15}}{\partial x} [V_1^2 + 2 \Delta \bar{V}_1 V_1 + \Delta \bar{V}_1^2 + \Delta \bar{V}_2^2] \\ + \frac{\partial C_m}{\partial x} \Delta \bar{V}_2 [V_1 + \Delta \bar{V}_1]$$

$$\left(\frac{\partial C_{15}}{\partial x} = \frac{\partial C_{25}}{\partial x} \quad \text{for isolated spheres} \right)$$

$$\Delta F = F_1 - F_2$$

$$= \frac{1}{2} \frac{\partial C_{15}}{\partial x} [2 \Delta \bar{V}_1 V_2 + \Delta \bar{V}_1^2 + \Delta \bar{V}_2^2] \\ + \frac{\partial C_m}{\partial x} \Delta \bar{V}_2 [V_1 + \Delta \bar{V}_1]$$

Computations are more convenient if the spheres be reduced to one centimeter dimensions; then for an original unit charge and unit potential on the hot sphere and additional charges due to the plane of g_1 and g_2 in the hot and grounded spheres respectively:

$$\Delta F = \frac{1}{2} \frac{\partial C_{1s}}{\partial x} [2g_1 + g_1^2 + g_2^2] + \frac{\partial C_m}{\partial x} g_2 [1 + g_1]$$

$\frac{\partial C_{1s}}{\partial x}$ and $\frac{\partial C_m}{\partial x}$ are obtainable to more than needed accuracy from the tables computed by Lord Kelvin and given in Table II; g_1 and g_2 can be obtained quite accurately almost directly and the accuracy improved by applying corrections. The first direct values of g_1 and g_2 are obtained by noting from Table II the total charges in each sphere for unit potential and a quarter diameter spacing. The sum of these charges (called Q) with the proper sign is divided by 24 and the first value $g_1 = g_2$ is obtained. This assumes that 24 radii is so great compared to 1 radius that all the charges in the image spheres may be assumed at the same point 24 radii away. This may be improved by adding separately the effects of the principal charges (which are comparatively easily obtained) in their true positions; then g_1 and g_2 differ very slightly. (This last step introduces too small a change to warrant its use.) Q as determined by this

Table II

Gap Setting in Per Cent Diameter	$\frac{C_{s1}}{\text{radius}}$	$-\frac{C_m}{\text{radius}}$	$\frac{1}{2} \frac{\partial C_{s1}}{\partial x}$	$-\frac{\partial C_m}{\partial x}$
5	1.58396	.88175	1.13844	2.34878
10	1.43131	.72378	.52852	1.12700
15	1.34827	.63395	.32917	.72714
20	1.29316	.57202	.23159	.52928
25	1.25324	.52537	.17432	.41260
30	1.22218	.48819	.13696	.33574
35	1.19755	.45746	.11082	.28180
40	1.17738	.43140	.09174	.24146
45	1.16056	.40886	.07720	.21052
50	1.14629	.38908	.06592	.18598
55	1.13404	.37151	.05693	.16608
60	1.12340	.35571	.04963	.14962
65	1.11410	.34150	.04363	.13582
70	1.10588	.32852	.03863	.12406
75	1.09859	.31663	.03441	.11394
80	1.09208	.30569	.03084	.10514
85	1.08623	.29557	.02775	.09744
90	1.08095	.28617	.02509	.09062
95	1.07617	.27742	.02278	.08458
100	1.07182	.26942	.02075	.07916

All values in electrostatic units

Table II (continued)

C_s and C_m are the self and mutual capacitances respectively and $\frac{\partial C}{\partial x}$ denotes the rate of change of capacitance with respect to distance between centers. Hence, if the potentials of the two spheres are V_1 and V_2 , then the total charges in each, Q_1 and Q_2 , and the attractive force are given by:

$$Q_1 = C_s V_1 + C_m V_2$$

$$Q_2 = C_s V_2 + C_m V_1$$

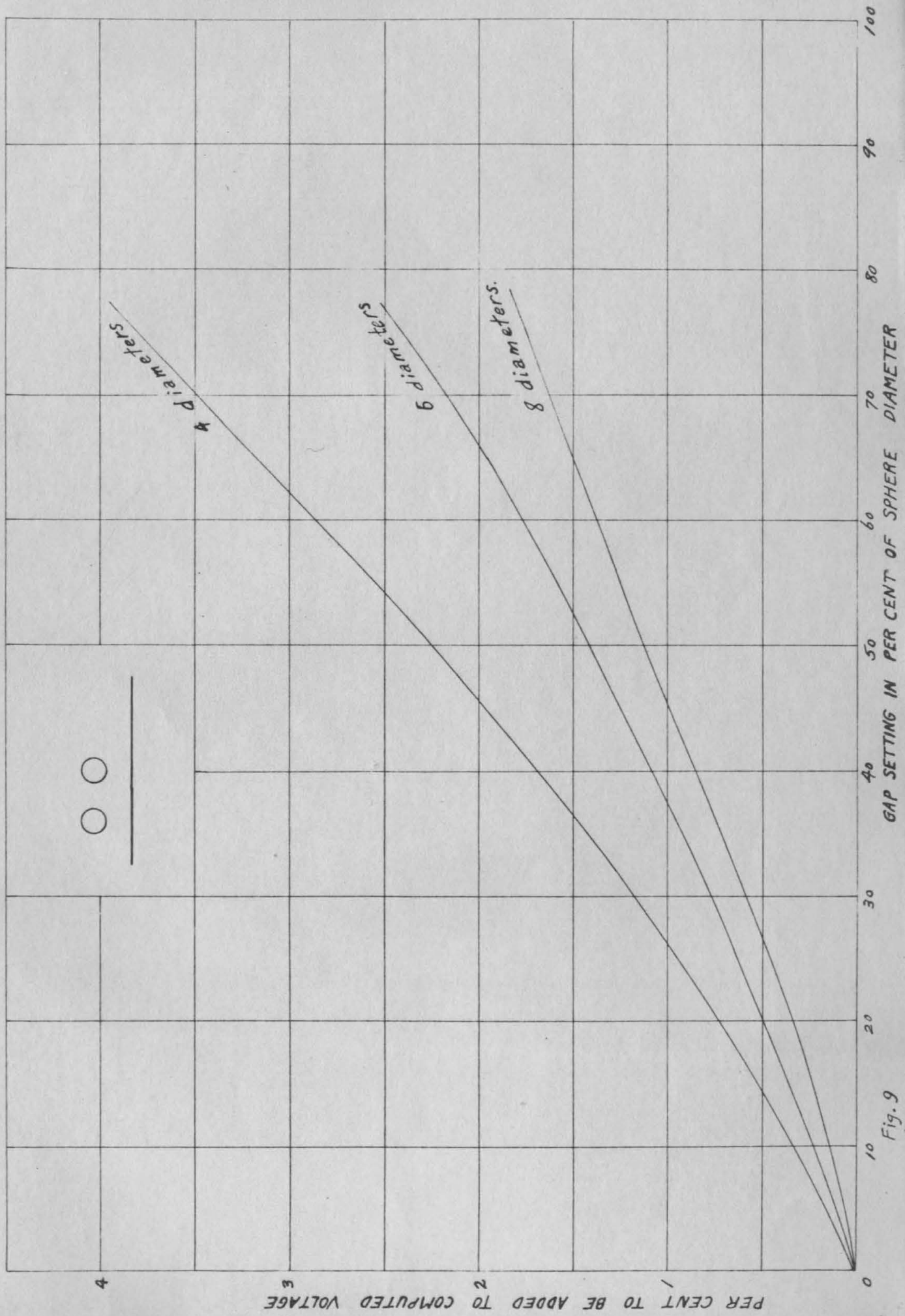
$$F = \frac{1}{2} \frac{\partial C_s}{\partial x} [V_1^2 + V_2^2] + \frac{\partial C_m}{\partial x} V_1 V_2$$

first step should be increased because of the presence of the new charges, g_1 and g_2 , in the image spheres as well as in the original spheres. The new and better value to replace g_1 is given then by $g_1' = g_1 + \frac{2g_1 Q}{24}$. (This expression is in units of one centimeter radius spheres. That is, Q is a number resulting from the addition of charges in two one-centimeter spheres, one raised to unit potential and the other grounded. If both spheres should receive a potential of g_1 then the total charge is not $Q \times 1 = Q$ but rather $Q \times g_1 \times 2 = 2g_1 Q$) A still better value to improve g_1 is

$$g_1'' = g_1 + \frac{2g_1 Q}{24} + \frac{2 \times 2g_1 Q^2}{24 \times 24}$$

For the case of 6 diameters to a parallel ground plane g_1'' was used and ΔF computed as outlined for spacings from 5 to 75 per cent diameter. The results stated in terms of correction to voltage rather than force are shown by the curves of Figure 9. The sign of ΔF is always opposite to F , which means that a ground plane always decreases the force for a given voltage. ΔV in per cent is one-half ΔF because V is proportional to the square root of force.

Computations also have been made for two other spacings as shown in Figure 9. These three curves should give the correction factor to a few per cent of its true value, the curve for 8 diameters being the most accurate and that for 4 diameters the least



accurate. Account was taken in each case of the effect termed factor 2 on page 34 - the direct action of the charges in the image spheres. This contribution, even for the 4 diameter clearance case, was never more than several per cent of the correction factor, but it was taken into account. These curves give the effect of planes on previously isolated spheres, not the effect of planes on spheres with shanks. Though the shanks themselves cause a disturbance, they do not influence the separate effect of the ground planes more than to change the ordinates of the curves Figures 9 and 10 by a few per cent, the order of magnitude of the disturbance due to shanks acting alone.

The same assumptions described for the case of a plane parallel to the line of centers may be applied but with less accuracy to the case of a plane perpendicular to the line of centers. Here again the disturbance may be considered as made up of the same two factors: (1) the added contribution due to the new charges which appear in the original spheres; (2) the direct action of all the charges in the image spheres on all the charges in the original spheres. Factor 2, which previously was unimportant now becomes about one-fourth of the total disturbance in the case of a plane of five diameters clearance to the closest and grounded sphere. This, of course, is due to the fact that the charges in the image spheres act on the charges in the principal

spheres directly along the line of centers instead of at a large angle as in the previous case. However, factor (2) may be computed with good accuracy by the same procedure outlined for computing this part of the disturbance in the previous case.

Factor (1) cannot be so accurately computed as in the case of a parallel plane by the same method used there, because what were termed the "center" images in the previous case, while they still fall at the same small distance from the center (for the same clearance to ground), occur now along the line of centers. Previously, the small displacement was almost perpendicular to the line of centers and resulting images were practically unchanged. But the change of succeeding images from what they would be if the "center" charges were exactly at the center, is in this case only of the order of one to two per cent, and factor (1) is only 75 per cent of the total disturbance for a clearance of five diameters to ground, so the curve for this clearance in Figure 9 is still believed to be accurate to certainly better than 10 per cent. The curve for 10 diameters to ground should be more accurate. For this clearance, item 2 becomes about 15% of item 1. The contribution of factor (2) opposes that of factor (1), hence the ordinates of Figure 10 are smaller than those in Figure 9 for like clearances.

From the results and discussion presented, it is clear that very good approximations may be obtained for the case of

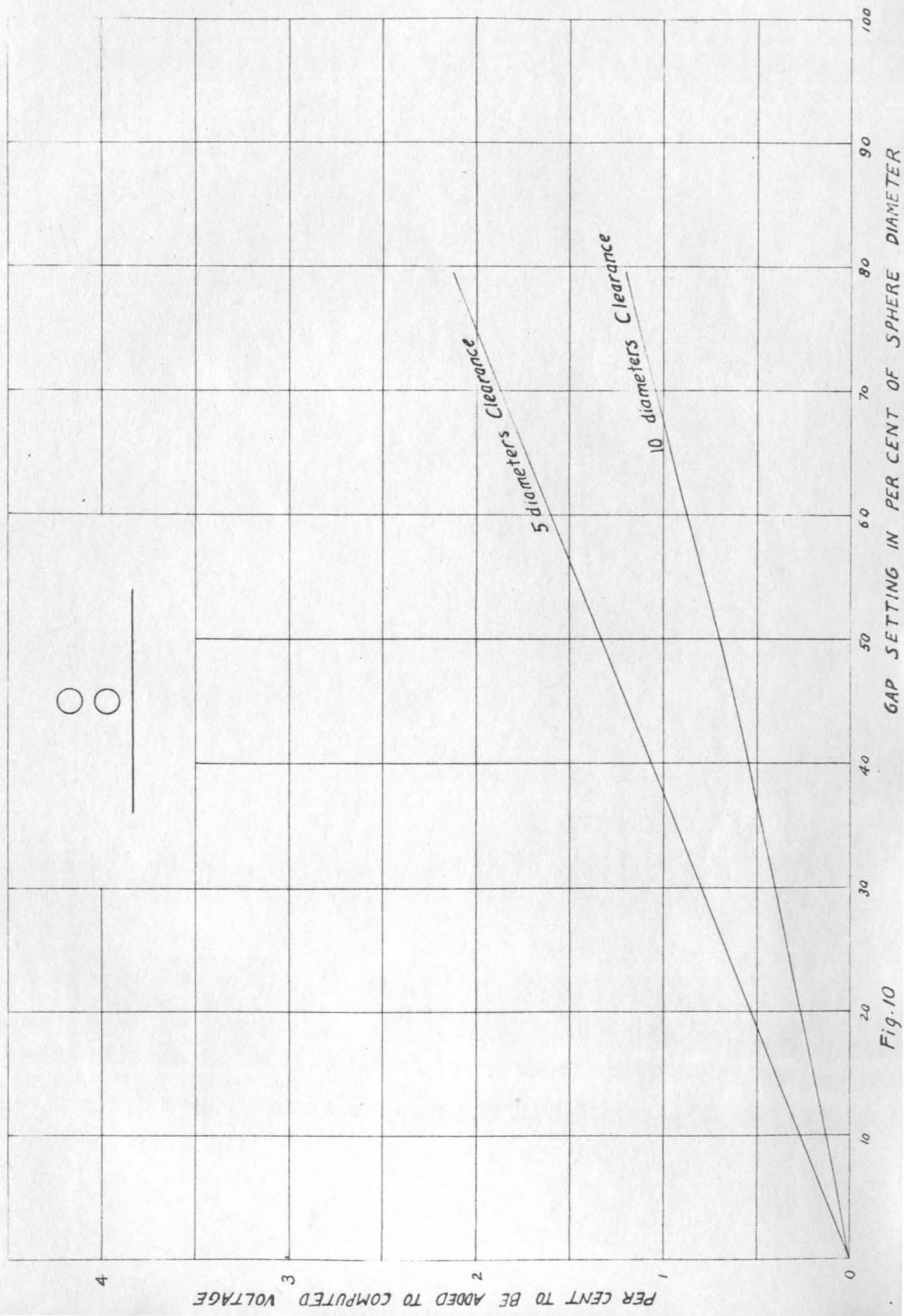


Fig. 10

combinations of planes, such as an intersecting floor and wall. In fact, for the clearances such as those available in the California Institute of Technology laboratory, an excellent estimate may be obtained by the repeated use of the curves (Figures 9 and 10) already computed. The net effect will be less than the sum of the effects due to the individual planes because of the presence of new "diagonal" image spheres with their images having a sign opposite to the image spheres already investigated.

To find the effect of a corner as pictured in Figure 11:

1. Find the disturbance due to spheres A for a clearance d_A from Figure 9.
2. Find the disturbance due to spheres B for a clearance d_B from Figure 10.
3. For a clearance d_C take a weighted mean of the disturbances given by Figure 9 and Figure 10, remembering that the factor (2) previously discussed, contributes only a few per cent to the ordinate of Figure 9, but about 25% of the ordinate in Figure 10 for the 5 diameter clearance and about 15 per cent of the ordinate in Figure 10 for the 10 diameter clearance case.
4. Add the effects found in (1) and (2) and subtract the effect found in (3) for the final correction factor.

For more accuracy, the change in the total charge in the spheres A and B due to the presence of C must be considered.

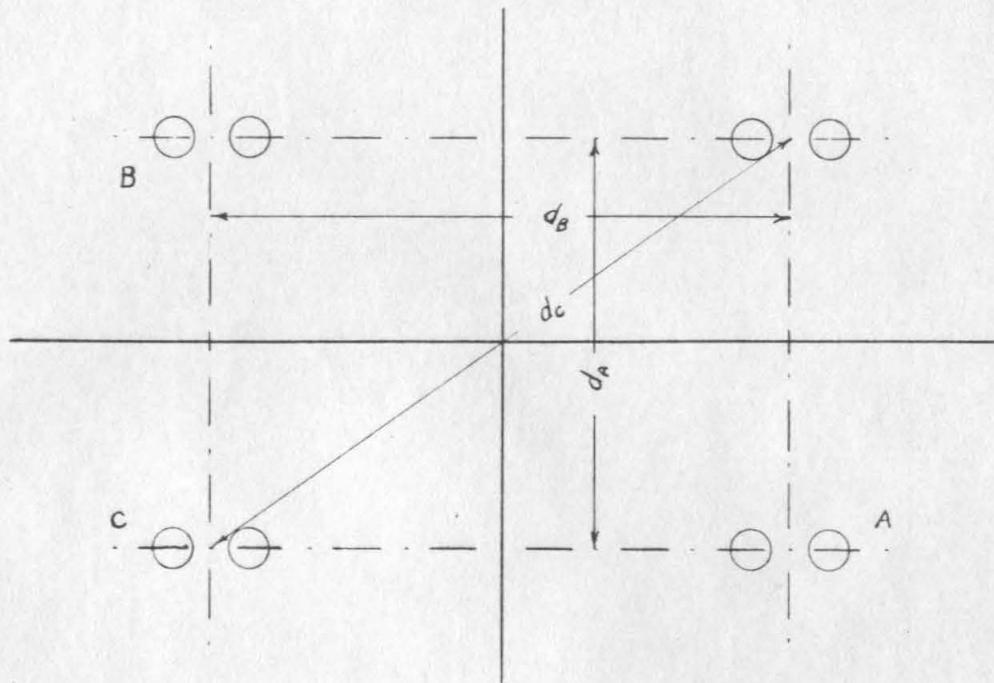


Fig. 11

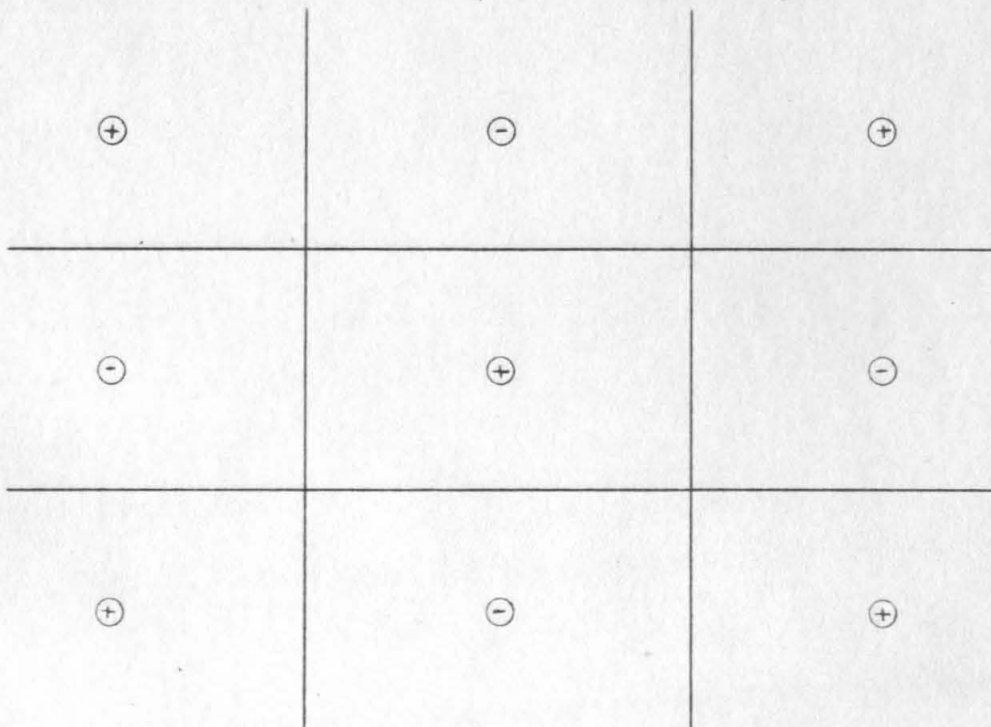


Fig. 12

This means that the charge thrown into the centers of the original spheres (termed g_1 in the preceding discussion and modified to g''_1) must again receive a small correction. Also, the effect of the diagonal image spheres should be computed according to the scheme used for arriving at the curves in Figures 9 and 10 rather than estimated by taking a mean of the readings from these curves. For clearances to walls and floors which exist in the laboratory, these refinements are unnecessary.

In the case of an angle in which both planes are parallel to the line of centers (such as a floor and side wall of the laboratory), only the curves of Figure 9 need be used and the error of approximation due to taking a mean of ordinates obtained from figures 9 and 10 does not appear.

The method may be applied to give an estimate of the correction factor to be applied when the spheres are enclosed by a grounded box-like structure. Here the number of image spheres becomes infinite. The closest of these image spheres are pictured in figure 12, the predominant sign of the charge in each pair of spheres being indicated. There are countless other spheres not shown which result from imaging each sphere into each plane. The procedure should be to find, as explained, the effect of the closest image spheres. The effect of spheres in a larger radius from the original spheres as center may be added in steps. The contributions will diminish in magnitude and alternate in sign

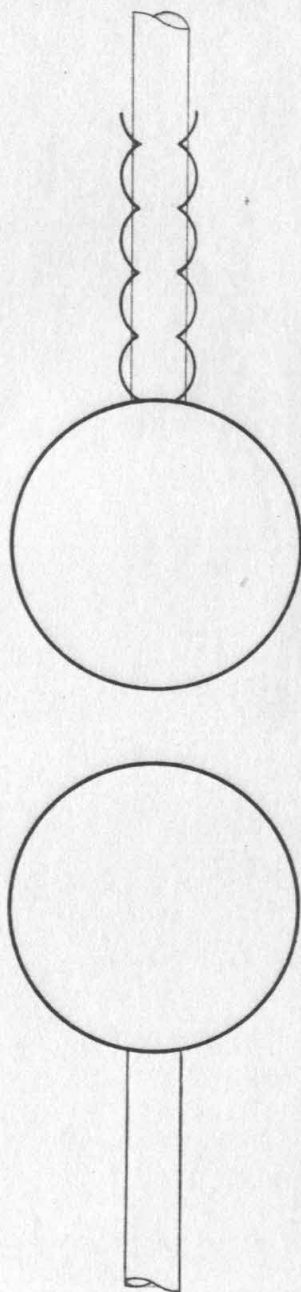


Fig. 13

so an approximate sum ought to be computable without difficulty. As more and more spheres are considered, refinement could be introduced by correcting the total charge in each sphere.

The scheme of introducing point charges along the shanks of the spheres to give an indication of the contribution to the force due to the presence of the shanks has already been mentioned. To arrive at the value of these charges, fictitious shanks made up of intersecting spheres were considered. (See Figure 13.) The correct charges necessary to keep these spheres at proper potential in the face of the spheres (large and small) themselves were sought. The procedure was as follows:

1. The value of the potential behind each sphere for various distances was computed from the principal charges in the original spheres.

2. This potential was neutralized by an image charge in each small shank sphere. In the case of the hot shank, an additional charge was placed at the center of each shank sphere to place it at the same potential as the large hot spheres.

3. The process of imaging every new charge into every sphere was carried on for a few steps.

4. A check was made by evaluating the potential (by summing charge over distance) at points along the shank. A trial and error process was used to improve the values of charges.

5. When the above process yielded a fairly smooth shank of proper dimensions, the contribution to force was computed.

The calculations were made for three cases: (1) 90 centimeter shanks of approximately $5\frac{1}{2}$ centimeters diameter - the diameter of the shanks in the laboratory - for a gap spacing of 25 centimeters; (2) 200 centimeter shanks for a gap spacing of 25 centimeters; (3) 200 centimeter shanks for a gap spacing of 50 centimeters.

In the 90 centimeter case, twenty charges spaced $4\frac{1}{2}$ centimeters apart were used on each shank. (This spacing of charges was chosen because at this distance between centers, two spheres of 6 centimeters diameter will intersect at approximately 90 degrees. The infinite series of images appearing for intersecting spheres reduces to a single charge for this angle of intersection.) The result showed that the shanks increased the force for a given voltage, as expected from simply a consideration of the fields. A voltage as computed by assuming isolated spheres should be too high by .6 per cent.

In the 200 centimeter case, forty charges were used on each shank. These are listed in Table III. The calculations showed 1.5 per cent error in voltage computation for the 25 centimeter gap spacing and 3 per cent error for the 50 centimeter gap spacing. From a first consideration it may seem that an increase from

Table IIIShank Charges

(Potential of hot sphere is taken as .02)

Distance from Sphere cm.	Hot Shank		Grounded Shank	
	25 cm. Gap	50 cm. Gap	25 cm. Gap	50 cm. Gap
5	.002	.003	.0008	.0006
10	.003	.004	.0009	.0007
15	.004	.004	.0010	.0008
20	.005	.005	.0011	.0009
25	.005	.005	.0012	.0010
30	.005	.005	.0013	.0010
35	.006	.006	.0013	.0011
40	.006	.006	.0013	.0010
45	.006	.006	.0013	.0008
50	.007	.007	.0013	.0008
55	.007	.007	.0013	.0010
60	.007	.007	.0013	.0010
65	.007	.007	.0013	.0011
70	.007	.007	.0013	.0011
75	.007	.007	.0013	.0011
80	.007	.007	.0013	.0010
85	.007	.007	.0013	.0011
90	.007	.007	.0013	.0011
95	.007	.007	.0012	.0011
100	.008	.008	.0012	.0008

Table III (continued)

Distance from Sphere cm.	Hot Shank		Grounded Shank	
	25 cm. Gap	50 cm. Gap	25 cm. Gap	50 cm. Gap
105	.008	.008	.0012	.0008
110	.008	.008	.0013	.0011
115	.009	.009	.0013	.0011
120	.009	.009	.0013	.0011
125	.008	.008	.0013	.0011
130	.008	.008	.0013	.0011
135	.008	.008	.0013	.0011
140	.008	.008	.0012	.0010
145	.008	.008	.0011	.0009
150	.008	.008	.0011	.0009
155	.008	.008	.0012	.0010
160	.009	.009	.0013	.0011
165	.010	.010	.0014	.0012
170	.011	.011	.0015	.0013
175	.012	.012	.0016	.0013
180	.014	.014	.0017	.0014
185	.016	.016	.0018	.0015
190	.018	.018	.0018	.0015
195	.018	.018	.0018	.0017
200	.019	.019	.0018	.0017

.6 per cent to 1.5 per cent error upon the addition of another 110 centimeters of shank length indicates an error for the much longer shank of the laboratory of much more than 1.5 per cent. However, further study showed that, because of the shielding effect of the hot sphere, the effect of added shank when the shank is less than 100 centimeters is very important; when the shank is greater than 100 centimeters the effect of added length is to contribute a rapidly decreasing increment. These conclusions were arrived at by computing the effect of a small sphere at the same potential as the hot sphere and placed at different positions from zero to 300 centimeters behind the hot sphere. From the results obtained it is believed that from 1.5 to 2.5 per cent is the order of magnitude of the correction due to shanks for a 25 centimeter gap spacing; 3.25 to 4.25 per cent is the error for 50 centimeter spacing.

From the curves of Figures 9 and 10 and a knowledge of the various clearances to ground existing in our laboratory (Figure 2) it is expected that the effects of grounds and shanks must very nearly cancel each other at 25 centimeters spacing. It is also concluded that the effects are almost equal and opposite over the whole range of spacings used in the sparking voltage measurements on the 50 centimeter spheres; the maximum spacing used was 35 centimeters. The curves of Figure 3, giving tertiary volts against kilovolts computed from force, are seen to consist of broken lines.

These lines do not connect smoothly because the gap setting of the sparkless spheres is different for each range of voltage. Some of this difference is due to an actual change in the relation between the secondary and tertiary terminal voltage with the change in capacity load and also a change in the voltage drop through the series water resistor. Some of this deviation, which is as much as 1.5 per cent between the curves for 25 and 35 centimeters gap spacing, may also be due to the variation with spacing of the disturbance due to grounds and shanks. On the basis of this and previous conclusions, this figure of 1.5 per cent was quoted in stating the limit of accuracy of the test points of Figures 5, 6, and 7 over the range between 40 and 50 centimeters spacing of the sparking spheres.

Conclusions - The Sparkless Sphere Gap

Voltmeter as an Absolute Standard

In Appendix A are listed ten very apparent and definite advantages of the sparkless sphere gap voltmeter over the conventional sparking spheres. These advantages were seen from the first work on the force meter. Further merits of this method may now be stated as a result of the research described in this thesis. These factors make the sparkless spheres worthy of serious consideration as a new A. I. E. E. standard method for high voltage measurement.

1. There is only one correction that need be applied to a voltage computed from force assuming isolated spheres. This correction factor, occasioned by the presence of grounds and shanks, is a constant for any set-up and can be determined with excellent accuracy.

2. The influences of floors, walls, shanks, etc. are small and opposite in direction. Thus, for a practical installation in a high voltate laboratory, the meter may be said to be an absolute standard.

3. For any spacing of the sparkless spheres, the force varies as the square of the voltage. This characteristic may be used to find the calibration of the instrument over its whole range of spacings once it has been found for one spacing by

comparing force readings at different spacings on constant voltage.

4. The present standard, the sparking spheres, gives readings which vary. The sparking voltage is not computable from the present inadequate theory; the influence of various factors not being known and completely understood, it is not possible to obtain a calibration curve which a given gap may be expected to duplicate. The sparking spheres are thus useful as a secondary, approximate means for measuring voltage. For a primary standard the non-sparking spheres are superior.

The Sparkless Sphere Gap Voltmeter

Sphere gaps may be used for measuring very high voltages by holding them at a separation greater than spark-over distance, and measuring the force between them. The voltage may then be calculated easily and accurately from well known electrostatic principles. Among the advantages of this type of measurement over the conventional spark-over methods are: no corrections are necessary for air temperature, humidity, and pressure; effective voltage values are given; measurements are not erratic; and numerous difficulties in taking measurements are avoided.

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THE sphere gap may be used as a Kelvin electrostatic type of voltmeter rather than a spark gap voltmeter, if the spheres are mounted so that the forces between them, due to applied differences of potentials, can be measured; and the authors of this paper believe the data presented herein indicate greater convenience and accuracy for sphere gaps thus used than can be obtained with gaps used in the conventional manner. The reasons for this statement are:

1. Accurate methods are available for calculating the relation between potential differences of 2 isolated spheres and the forces due to these potential differences when sphere dimensions, spacing, and the dielectric constant of their ambient medium are known.
2. Since the dielectric constant of air is not appreciably affected by temperature, air density, or humidity, no corrections for these factors are necessary.
3. The tests showed complete freedom from erratic readings so noticeable in the spark gap voltmeter.

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. summer convention, Ithaca, N. Y., June 24-28, 1935. Manuscript submitted Feb. 25, 1935; released for publication Apr. 11, 1935.

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1. For all numbered references see list at end of paper.

4. Test planes of large size placed as near as possible to the spheres without causing sparking from spheres to plane had little influence on the readings.

5. The sensitivity of equipment used is such that a change in force of $\frac{1}{2}$ gram or less can be detected easily, and forces from 100 to 400 grams were available in the tests. This degree of sensitivity is such that the instant of contact of the spheres at zero spacing can be noted by a movement of the free sphere more readily than by noting the electric contact or any other means known.

APPARATUS USED

The apparatus used in the experiments is shown in figures 1, 2, and 3. The only large spheres available at California Institute of Technology for making these tests were a pair of 100 centimeter cast aluminum spheres made at the same time and as exact duplicates of the pair used by Carroll and Cozzens¹ in their tests; and these spheres show, on spark-over tests, the same erratic performance as reported by them. The right-hand sphere is supported by a rigid insulating frame suspended from the roof structure of the laboratory and is mounted in such a way as to provide for adjustment of the sphere gap for spacings from 0 to 150 centimeters. Changes in the sphere gap setting are made by means of a motor-driven mechanical system, the motor being placed on the floor and attached to, but insulated from, the sphere driving mechanism by a long rope belt. The left-

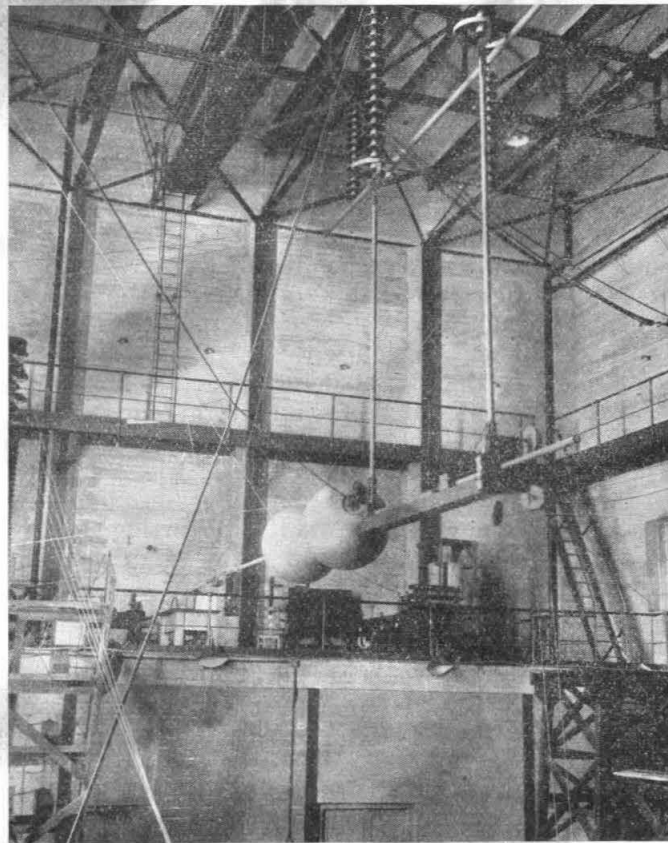


Fig. 1. A view of the sphere gap and other principal pieces of apparatus comprising the sparkless sphere gap voltmeter. The sphere in the foreground is the hot sphere. The measuring apparatus, shown in more detail in figure 3, may be seen at the far end of the shaft of the grounded sphere

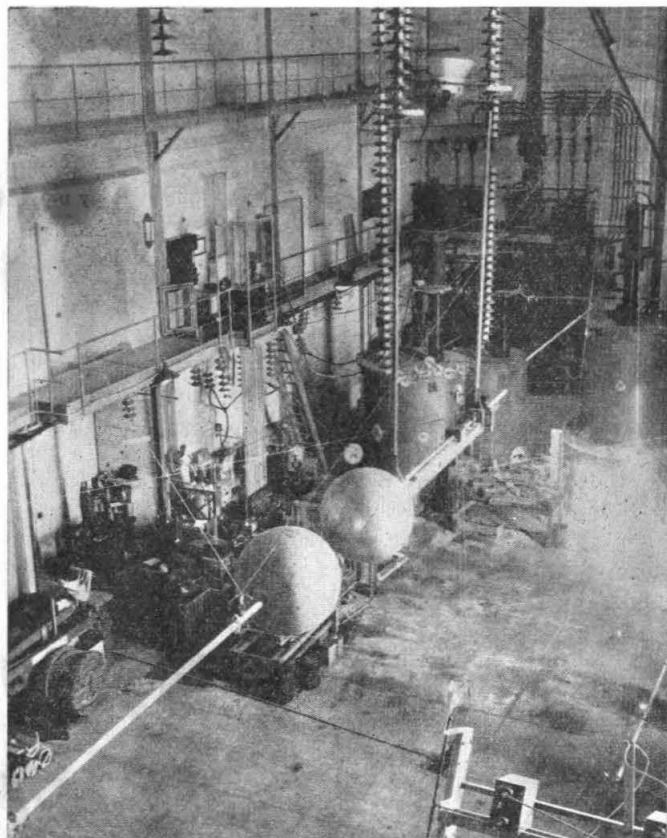


Fig. 2. Another view of the apparatus, showing the relation of the sphere gap to the other equipment in the laboratory, and showing the large clearance available around the gap. The grounded sphere is in the foreground of this view. Note the long V suspension ropes

hand or free sphere is suspended by 4 ropes arranged in 2 pairs, each pair forming a letter V, the apex of which is attached to the shaft supporting the free sphere. This suspension with the upper ends of the V attached to the roof structure provides horizontal force components which prevent lateral motion of the sphere and at the same time allows very free motion along the axis through the center of the 2 spheres. In the tests made, this free sphere was grounded by very flexible connections from its supporting shaft to ground. Mounted on this supporting shaft near the end away from the sphere, is a fine wire which serves as a pointer, any motion of which may be observed through a cathetometer telescope. Also at this end of the shaft, there is attached a cord which passes over a bicycle wheel used as a pulley and supports a weight pan on which weights may be placed to balance the pull between the spheres. With this arrangement the natural damping was sufficient to prevent oscillations.

A water tube resistance of about 2 megohms, made of $\frac{3}{4}$ inch garden hose through which tap water is run for cooling, is connected in the circuit between the transformers and the insulated sphere to prevent burning of the spheres when the gap flashes over. This resistance is kept insulated from ground by having the flowing water fall into a funnel

at the top of the hose and from the outlet into a tank on the floor. The hose is protected from corona burns where necessary by pie tins used as disk shields spaced about 18 inches apart.

TAKING TEST READINGS

In the work to date, readings of voltage have been made by setting the sphere gap just above spark-over distance for each voltage measured and enough weight put on the pan to balance the pull on the spheres and keep the free sphere from moving, as noted by the cross hair. With voltage held constant by means of a voltmeter in the transformer tertiary "volt" coil, the sphere gap distance was then decreased until the gap flashed over and the length of gap at the time of flashover noted. The potential difference for this spacing, as indicated by use of the A.I.E.E. sphere gap curve, may then be compared with the voltage calculated by the equation

$$V = 9,405\sqrt{F/S}$$

where F is the weight in grams on the pan, and S is the spacing factor shown in figure 8. Results of these measurements are shown in figure 4, typical data for which are given in table I.

V in the above equation is the effective value of voltage; hence, to compare it with the A.I.E.E. sphere gap curve, the wave form must be known. No difficulty was encountered at any time in duplicating readings showing the relation between the force readings as determined by the weights on the pan and the voltmeter readings as obtained at the "volt" coil; and the sensitivity and precision of the force measurements was found to be much better than the degree of accuracy obtainable in reading the voltmeter and setting the voltage control regulator. Figure 5 shows two 2 wave forms, the lower, the input volt wave to the testing transformer, and

Table I—Typical Test Data for Voltage Measurement With One-Meter Spheres

Temperature "T" = 13 degrees centigrade
Barometric Pressure "b" = 747 mm of mercury
 $\delta = \frac{0.392 b}{273 + T} = 1.023$

Reading Number	Gap Setting for Force Measurement	Force in Grams	Tertiary Voltmeter Reading	Spark-Over Distance	Spark-Over Distance Corrected	S	Calculated Voltage*
2...35 cm...	346.1...	150	...	31.1 cm...	32.1 cm...	0.11082...	526,000
3...40 cm...	376.6...	170	...	36.6 cm...	37.9 cm...	0.09174...	603,000
5...50 cm...	354.7...	190.5	...	43.8 cm...	45.4 cm...	0.06592...	690,000
7...55 cm...	362.5...	202	...	51.7 cm...	53.7 cm...	0.05693...	750,000
10...65 cm...	338.9...	220	...	62.7 cm...	65.4 cm...	0.04363...	829,000
13...70 cm...	309.1...	224	...	65.8 cm...	68.4 cm...	0.03863...	840,000

* Calculated by equation: $V = 9,405\sqrt{F/S}$

the upper one, the output voltage wave. The output wave was recorded by using a water tube resistance of about 5 megohms in series with an oscillograph; one end of the resistance being connected to the supporting shaft of the ungrounded sphere, and the other end connected through the

oscillograph element to ground. An analysis of the output wave, using 50 ordinates, shows its root mean square value to be 99.25 per cent that of a true sine wave having the same crest value. Figure 5 shows the wave form is practically a sine wave.

RELATION OF FORCE TO VOLTAGE

The use of inverted images for calculating the force between 2 spheres may perhaps be more readily understood by considering the case for a point charge in the presence of a grounded conducting sphere isolated in space.^{2,3,8}

When no other body is present, the potential at any point a distance r from the point charge q is $V = q/r$ which equation satisfies the one boundary condition that the potential be zero at an infinite distance from the charge. When the grounded sphere is introduced, the equation for potential at any point in space must satisfy the additional condition that the potential is zero at every point on the sphere. Simple geometry provides the equation for V which will satisfy these conditions.

In figure 6, o is the center of a sphere of radius a ; charge q is a distance f from o ; p' is a point on the line op at a distance d from o ; s is any point on the surface of the sphere. From the figure,

$$r = \sqrt{a^2 + f^2 - 2af \cos \theta}$$

and

$$r' = \sqrt{a^2 + d^2 - 2ad \cos \theta}$$

If d is made equal to a^2/f :

$$\begin{aligned} r' &= \sqrt{a^2 + \frac{a^4}{f^2} - 2 \frac{a^3}{f} \cos \theta} \\ &= \frac{a}{f} \sqrt{a^2 + f^2 - 2af \cos \theta} \end{aligned}$$

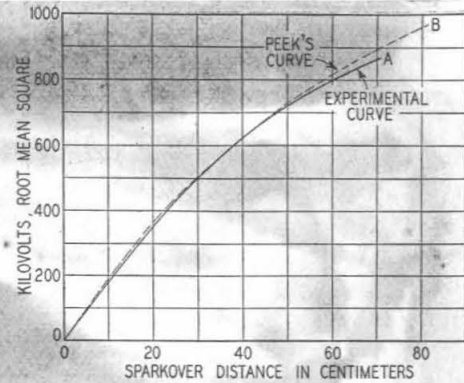
and

$$\frac{r'}{r} = \frac{a}{f} \quad \text{or} \quad \frac{1}{r} = \frac{a}{f} \cdot \frac{1}{r'}$$

With a charge q at point p and another charge $q' = -\frac{a}{f} \cdot q$ at point p' , the expression for potential at any point in space a distance r from p and a distance

Fig. 4. Relation between spark-over voltage (effective) and sphere gap setting at spark-over distances

Curve A from force measurement data
Curve B from Peek's values for 100 centimeter spheres



r' from p' would satisfy the condition that the potential be zero on the surface of the sphere; since

$$V = \frac{q}{r} + \frac{q'}{r'} = q \left[\frac{1}{r} - \frac{a}{f} \cdot \frac{1}{r'} \right]$$

and

$$\frac{1}{r} = \frac{a}{f} \cdot \frac{1}{r'}$$

everywhere on the sphere.

Point p' is the inverted image point of p and the above expression for potential completely defines the field outside the sphere. The field outside a grounded sphere due to an adjacent point charge is the same as the field caused by the original point charge and its image point charge. The image charge is always $(-a/f)$ times the original charge and is located a distance a^2/f from the center of the sphere.

This analysis can be extended to show the field about a sphere A at a potential V in the presence of another sphere B at ground potential. The field outside an isolated sphere A at a potential V is equal to that due to a charge $q_1 = aV$ at the center of the sphere, where a is the radius which for a sphere is equal to its capacitance. If, as in figure 7, another sphere B , of the same radius and grounded, is brought into the field with its center a distance c from the center of A , the field will be distorted. But, as has

been shown, a charge $q_2 = -\frac{a}{c} q_1$ at a distance d_2

$= \frac{a^2}{c}$ from the center of the grounded sphere B will,

together with the original charge q_1 in the equation, give zero potential over the surface of sphere B . This new expression, however, no longer satisfies the condition that the voltage be equal to V over the surface of sphere A and it is necessary to cancel the effect on sphere A of the added charge q_2 by placing its image charge q_3 at a distance d_3 from the sphere center. If q_3 is the inverted image of q_2 , that is, if

$q_3 = -\frac{a}{c-d_2} q_2$ and $d_3 = \frac{a^2}{c-d_2}$ the resulting potential

on sphere A due to q_2 and q_3 will be zero and the sphere potential will be the desired value V due to q_1 only. To keep B at zero potential under the influence of q_1 , q_2 , and q_3 , this new charge q_3 must be

imaged by a charge q_4 in B at a distance $d_4 = \frac{a^2}{c-d_3}$

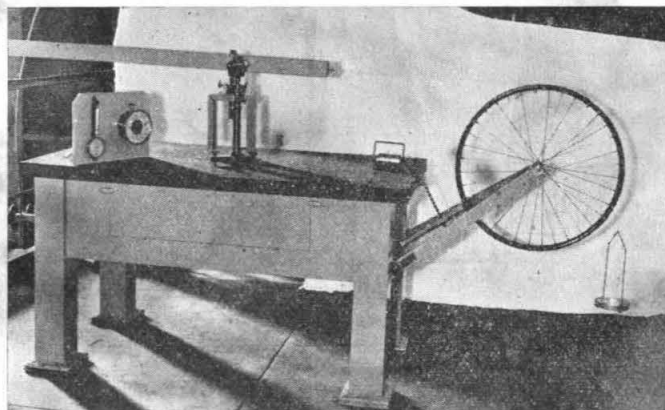


Fig. 3. A closeup view of the measuring apparatus

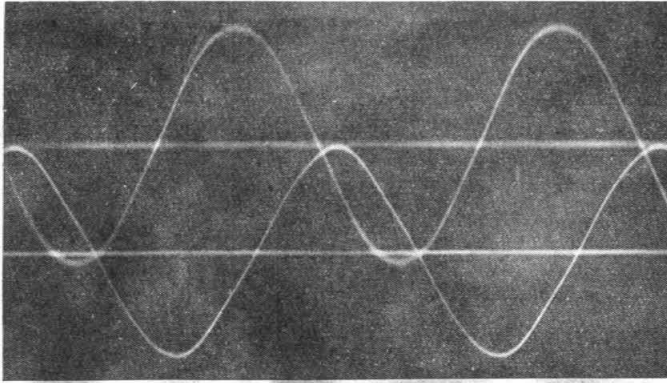


Fig. 5. Oscillogram of voltage, showing wave form

The upper curve shows the form of the voltage applied to the sphere gap
The lower curve shows the form of the voltage applied to the primary of the transformer

from the center of B . The value of q_4 is $-\frac{a}{c-d_3} q_3$ to satisfy boundary conditions. Thus a double series of images is set up, all those in sphere A having the same sign and all those in sphere B having the opposite sign to the original charge q_1 .

In general $q_n = -\frac{a}{c-d_{n-1}} q_{n-1}$ and $d_n = \frac{a^2}{c-d_{n-1}}$

The total force on sphere B is then the sum of the attractions on each charge in B due to every charge

Table II—Calculated Images

Radius "a" = 50 centimeters Spacing = 30 centimeters

Sphere A (Potential V)		Sphere B (Potential Zero)	
$q_1 = Va$	$d_1 = 0$ cm	$q_2 = -0.385$	$Va \dots d_2 = 19.2$ cm
$q_3 = 0.174$	$Va \dots d_3 = 22.6$ cm	$q_4 = -0.086$	$Va \dots d_4 = 23.2$ cm
$q_5 = 0.0378$	$Va \dots d_5 = 23.4$ cm	$q_6 = -0.0177$	$Va \dots d_6 = 23.45$ cm
$q_7 = 0.00833$	$Va \dots d_7 = 23.5$ cm	$q_8 = -0.00393$	$Va \dots d_8 = 23.55$ cm

in A . Since the attraction between 2 charges q_i and q_j a distance f apart is $\frac{q_i q_j}{f^2}$ the total force on B may be expressed as a summation:

$$F = \sum_{\substack{2 \\ \text{(even)}}}^{\infty} \sum_{\substack{1 \\ \text{(odd)}}}^{\infty} \frac{q_i q_j}{(c-d_i-d_j)^2}$$

Since c , even for very small gap settings, must be at least twice a , the n th image charge is much smaller than the $(n-1)$ th charge. The inclusion of more pairs of images contributes rapidly diminishing amounts to the total force. In any case it is necessary to take only as many images as are required to make sure the total force of all the neglected images will be less than the allowable error.

A calculation for a 100 centimeter sphere gap with a setting of 30 centimeters will illustrate the method. The first 4 pairs of images and the distance of each from the center of its respective sphere is given in table II.

The force, due to these 4 pairs of charges only, is then:

$$F = \frac{q_1 q_2}{[130-d_2]^2} + \frac{q_3 q_2}{[130-d_2-d_3]^2} + \frac{q_5 q_2}{[130-d_2-d_5]^2} + \frac{q_7 q_2}{[130-d_2-d_7]^2} + \frac{q_1 q_4}{[130-d_4]^2} + \frac{q_3 q_4}{[130-d_4-d_3]^2} + \frac{q_5 q_4}{[130-d_4-d_5]^2} + \frac{q_7 q_4}{[130-d_4-d_7]^2} + \frac{q_1 q_6}{[130-d_6]^2} + \frac{q_3 q_6}{[130-d_6-d_3]^2} + \frac{q_5 q_6}{[130-d_6-d_5]^2} + \frac{q_7 q_6}{[130-d_6-d_7]^2} + \frac{q_1 q_8}{[130-d_8]^2} + \frac{q_3 q_8}{[130-d_8-d_3]^2} + \frac{q_5 q_8}{[130-d_8-d_5]^2} + \frac{q_7 q_8}{[130-d_8-d_7]^2}$$

Which, for $q_1 = aV$ (a being 50 centimeters) reduces to, if V is expressed in statvolts:

$$F = 0.137 V^2 \text{ dynes}$$

In general, $F = SV^2$ dynes for V expressed in statvolts or; $V = 9,405 \sqrt{\frac{F}{S}}$ for V expressed in practical

volts, and F in grams. With charges 7 and 8 neglected, the constant S would be reduced about one per cent, and, as their contribution to the total force is less than half the contribution of charges 5 and 6, the error made by neglecting all charges beyond 8 cannot exceed one per cent. The values for S , correct to the 5th decimal place, given in table III were computed with enough image charges to give this accuracy for spacings from 0 to 100 centimeters. These tabulated values are plotted in figure 8, and were used in conjunction with the force measurements to calculate the voltages for the one-meter spheres.

Since the larger the force measured, the smaller the experimental error, the gap should be set as small as possible without permitting flashover for each measured voltage. For flashover gap settings, using F. W. Peek's data⁴ for the flashover of one-meter spheres, forces approximately those shown in figure 9 will be obtained.

Figure 10 shows for one-meter spheres the force as a function of voltage for sphere gap spacings of 15, 30, 50, 75, and 100 centimeters.

Fig. 6. Diagram representing a sphere and a point charge

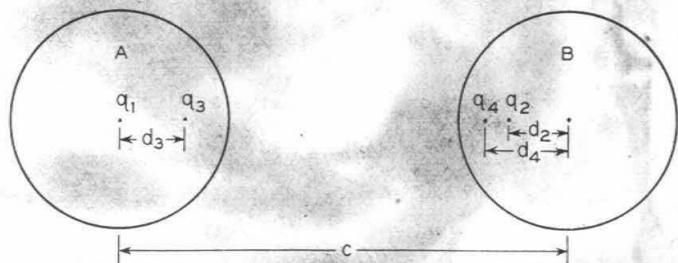
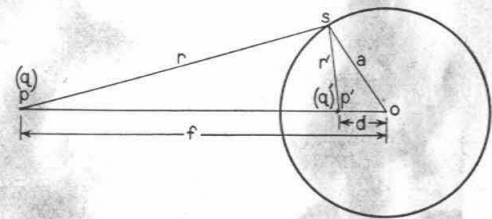


Fig. 7. Diagram representing 2 spheres, sphere A being at a potential V , and sphere B being at ground potential

INFLUENCE OF GROUND PLANES

In like manner, images may be used for the calculation of the influence of ground plane effects on the force between the spheres. The accurate calculation of such influence is possible, but laborious. To compute the change in the force caused by an infinite ground plane (for instance, a laboratory floor or wall), each charge of the original 2 infinite sets of images must be imaged to a position as far behind the plane as the charge lies in front of the plane. These 2 new sets of images must in turn be imaged back into the spheres following the law for sphere images presented above, each image being the origin of an infinite series of images in the spheres, and the procedure continued until the imaged charges become negligibly small. Consequently, the number of charges to be considered in calculating the force existing between the spheres is very much larger than for the case of isolated spheres.

Slide rule computations for the one-meter spheres set at 25 centimeters gap, were made of the disturbing effect of grounded infinite planes, one 20 feet from the gap, parallel to the line of the sphere centers, representing the laboratory floor; and the other 20 feet from the gap, perpendicular to the line of the sphere centers, representing an end wall. Since only an approximate indication of the disturbance was desired, image charges less than one per cent of the original charge were neglected; and charges located within a few centimeters of each other were grouped in a mean position when considering the forces exerted on them by charges several hundred centimeters distant. The result of these calculations indicated the error due to the assumption of isolated spheres to be of the order of 0.5 per cent. For the number of images considered in the computation, this is within slide rule accuracy. These calculations and the experiments with test planes to date, are believed to warrant the conclusion that for the test conditions in this laboratory, the disturbing effects are negligible for gap spacings up to at least 30 centimeters.

Should further work indicate the need for more complete corrections for ground planes, when the gap settings are large compared to the sphere diam-

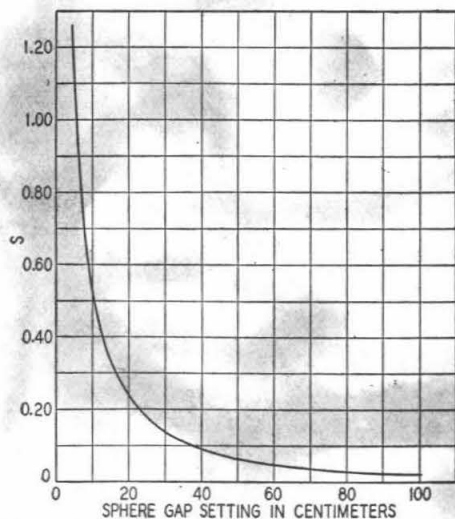
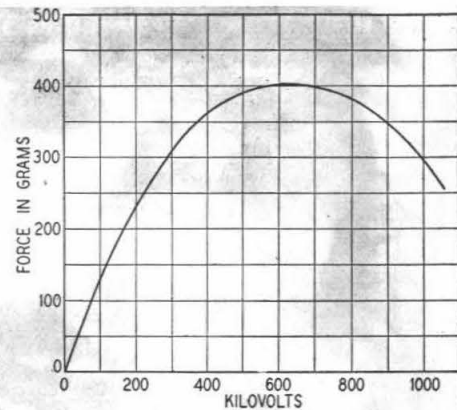


Fig. 8. Relation between sphere gap setting and the value of S used in the equation: $V =$

$$9,405\sqrt{F/S}$$

Fig. 9. Relation between force in grams and kilovolts with spheres set at Peek's values of sparking distances



eter, such corrections will be a matter only of more extensive calculations.

RÉSUMÉ

Curve A of figure 4 has been drawn as a mean curve through the test points without the points being shown, because, when all the points were shown, they made the curve, at the scale drawn,

Table III—Computed Value of the Spacing Factor S

(Sir W. Thomson—"Papers on Electrostatics and Magnetism," p. 96)

Sphere Gap Setting—Cm	S	Sphere Gap Setting—Cm	S	Sphere Gap Setting—Cm	S
0.....∞		35.....0.11082		70.....0.03863	
5.....1.13844		40.....0.09174		75.....0.03441	
10.....0.52852		45.....0.07720		80.....0.03084	
15.....0.32917		50.....0.06592		85.....0.02775	
20.....0.23159		55.....0.05693		90.....0.02509	
25.....0.17432		60.....0.04963		95.....0.02278	
30.....0.13696		65.....0.04363		100.....0.02075	

look like a broad ragged line. This is due to the well-known erratic behavior of the sphere gap when used to measure voltage by the spark-over method. The deviations in the test carried out by the authors had the following range from curve A: at 20 centimeters spacing, from about 1 per cent below to 1 per cent above; at 40 centimeters spacing, from about 2 per cent below to 1 per cent above; at 60 centimeters spacing, from 6 per cent below to 2 per cent above; at 70 centimeters spacing, from 6.5 per cent below to 1 per cent above.

It is interesting to note that curve A is practically coincident with the curve shown by Meador⁵ in his figure 8.

Attention is directed to the form of the curve in figure 9 which shows that the maximum force between the spheres, when they are used as close as possible to spark-over voltage settings for the force measurements, occurs at a spacing of about 40 centimeters which is just the minimum spacing at which trouble from erratic sparking commences. Perhaps this is an indication that difficulties will be encountered if 100 centimeter spheres are used at spacings above this value for voltage measurements by the spark-over method.

The advantages of using force measurements

rather than spark-over distances for sphere gaps are:

1. No corrections are necessary for temperature, humidity, and barometric pressure, the only air characteristic of influence being the dielectric constant.
2. For any given spacing the relation between force and voltage may be calculated accurately from fundamental electrostatic theory without the use of any empirical data.
3. From (2) this method appears to have value as an absolute standard.
4. Adjacent bodies, floors, walls, etc., have at least no greater effect on force measurements than on spark gap measurements and their influence on force measurements is subject to exact calculation.
5. In making voltage tests on apparatus, the use of force measurements permits continuous voltage application and avoids the well-known difficulties incident to the spark gap method, not the least of which are the oscillations sometimes set up.
6. No series resistance is required if care is exercised to keep the spheres at all times separated far enough to avoid spark-over.
7. Less care is needed in making the spheres capable of withstanding spark-over and in maintaining a highly polished surface free from dust, lint, etc.
8. Absence of polarity effects, and effects due to the state of gap ionization.
9. Applicable to continuous potential, and to alternating potential of any frequency, without any change in the force-voltage relation.
10. Readings are always root mean square or effective values regardless of wave form.

The authors request the interest and co-operation of other laboratories in checking the results obtained for spheres in air and in extending this method of

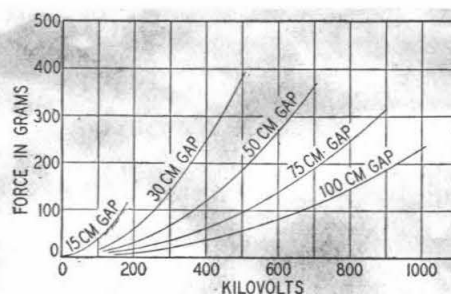


Fig. 10. Relation between force in grams tending to pull the spheres together and kilovolts (effective) for several gap settings (centimeters)

measurement to spheres of different diameter and to spheres mounted in dielectrics other than air, such as oil or other liquid dielectrics.

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Appendix B
Methods Previously Used
to Obtain Sparking Curves

- Peek⁴ - Peek used the tertiary coil turn ratio to calibrate spheres up to 25 centimeters diameter. He then computed what the maximum field at the spheres' surface would have been for isolated spheres at that same voltage; this field was assumed to be that which would cause breakdown for spheres of any size, and the sparking curves for larger spheres were computed.
- Meador⁶ - Meador used the tertiary coil turn ratio and made direct readings of the sparking voltages for the 50 centimeter spheres.
- Bellaschi⁵ - Bellaschi obtained curves of surge sparking voltage for both positive and negative polarity surges using a potentiometer and measuring beam deflections on oscillograms. The negative polarity surge curves are said to be the same for all size spheres as the power frequency curves. The basis for this is that a negative polarity curve for 25 centimeter spheres was seen to agree with a curve obtained some years ago on 60 cycles for 25 centimeter spheres by measuring the charging current to a capacitor.

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